Logarithmic and exponential functions

The derivative of $y = \ln x$ (1)

QUESTION **1**   Differentiate:

a $y = \log_e x$

b $y = \log_e 2x$

c $f(x) = \ln 6x$

d $y = \log_e(7x + 5)$

e $y = \ln (1 - 2x)$

f $y = \ln (5x + 3)$

g $y = \ln x^2$

h $f(x) = \ln x^5$

i $y = \ln x^9$

j $f(x) = \ln (x^2 + 5)$

k $y = \ln (3x^2 - 4)$

l $y = \log_e (x^3 - 7x^2)$

QUESTION **2**   Find the derivative of:

a $y = (\ln x)^2$

b $f(x) = \log_e (3x - 1)^2$

Logarithmic and exponential functions

The derivative of $y = \ln x$ (2)

QUESTION **1**   Find the exact value of $f'(e)$ if:

a $f(x) = \ln x$

b $f(x) = \log_e (2x - 1)$

c $f(x) = 3 \ln (x^2 + 1)$

QUESTION **2**   Differentiate:

a $y = x \ln x$

b $y = x^4 \log_e x$

c $\dfrac{\ln x}{x^2}$

d $\dfrac{x + 1}{\ln x}$

Logarithmic and exponential functions

The integral of $\dfrac{1}{x}$

QUESTION **1**   Find:

a $\displaystyle\int \frac{dx}{x}$

b $\displaystyle\int \frac{6}{x}\, dx$

c $\displaystyle\int \frac{3}{x+2}\, dx$

d $\displaystyle\int \frac{2x}{x^2+5}\, dx$

e $\displaystyle\int \frac{3x^2}{x^3-2}\, dx$

f $\displaystyle\int \frac{3}{3x-7}\, dx$

g $\displaystyle\int \frac{8x}{x^2-3}\, dx$

h $\displaystyle\int \frac{dx}{4x-1}$

i $\displaystyle\int \frac{7}{1-2x}\, dx$

QUESTION **2**   Find the exact value of:

a $\displaystyle\int_2^5 \frac{dx}{x-1}$

b $\displaystyle\int_1^e \frac{dx}{2x}$

c $\displaystyle\int_0^3 \frac{2x}{x^2+3}\, dx$

d $\displaystyle\int_2^4 \frac{3x^2+1}{x^3+x}\, dx$

Page 107   1   a $\dfrac{1}{x}$ b $\dfrac{1}{x}$ c $\dfrac{1}{x}$ d $\dfrac{7}{7x+5}$ e $\dfrac{-2}{1-2x}$ f $\dfrac{5}{5x+3}$ g $\dfrac{2}{x}$ h $\dfrac{5}{x}$ i $\dfrac{9}{x}$ j $\dfrac{2x}{x^2+5}$ k $\dfrac{6x}{3x^2-4}$ l $\dfrac{3x-14}{x^2-7x}$

2   a $\dfrac{2\ln x}{x}$ b $\dfrac{6}{3x-1}$

Page 108   1   a $\dfrac{1}{e}$ b $\dfrac{2}{2e-1}$ c $\dfrac{6e}{e^2+1}$ 2   a $1+\ln x$ b $x^3(1+4\log_e x)$ c $\dfrac{1-2\ln x}{x^3}$ d $\dfrac{x\ln x-x-1}{x(\ln x)^2}$

Page 109   1   a $\ln x + C$ b $6\ln x + C$ c $3\ln(x+2)+C$ d $\ln(x^2+5)+C$ e $\ln(x^3-2)+C$ f $\ln(3x-7)+C$

g $4\ln(x^2-3)+C$ h $\dfrac{1}{4}\ln(4x-1)+C$ i $-\dfrac{7}{2}\ln(1-2x)+C$ 2   a $\ln 4$ b $\dfrac{1}{2}$ c $\ln 4$ d $\ln 6.8$