Logarithmic and exponential functions

TOPIC TEST

Time allowed: 1 hour

Total marks = 100

SECTION I Multiple-choice questions

12 marks

Instructions • This section consists of 12 multiple-choice questions
- Each question is worth 1 mark
- Fill in only ONE CIRCLE
- Calculators may be used

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1 $2^3 \times 2^2 = ?$

(A) 2^5 (B) 2^6 (C) 4^5 (D) 4^6

2 $8^8 \div 8^2 = ?$

(A) 1^4 (B) 1^8 (C) 8^4 (D) 8^6

3 $7m^0 + 7^0 = ?$

(A) 1 (B) 2 (C) 7 (D) 8

4 $p^{-3} = ?$

(A) $\sqrt[3]{p}$ (B) $\sqrt{p^3}$ (C) $\dfrac{1}{p^3}$ (D) none of these

5 $x^{\frac{m}{n}} = ?$

(A) $\sqrt[n]{x^m}$ (B) $\sqrt[m]{x^n}$ (C) $\dfrac{x^m}{x^n}$ (D) none of these

6 $m^{-\frac{2}{3}} = ?$

(A) $\dfrac{1}{\sqrt{m^3}}$ (B) $\dfrac{1}{\sqrt[3]{m^2}}$ (C) $\dfrac{m^2}{m^3}$ (D) $\dfrac{m^2}{\sqrt[3]{m}}$

7 $\log_4 2 = ?$

(A) $\dfrac{1}{2}$ (B) 1 (C) 2 (D) 4

8 $2\log_a 3 - \log_a 2 = ?$

(A) $\log_a 7$ (B) $\log_a 4.5$ (C) $2\log_a 1.5$ (D) cannot be simplified

9 The value of e^2 correct to three decimal places is?

(A) 0.301 (B) 0.693 (C) 6.581 (D) 7.389

10 $\dfrac{d}{dx}(e^{2x}) = ?$

(A) e^{2x} (B) $2e^x$ (C) $2e^{2x}$ (D) $\dfrac{1}{2}e^{2x}$
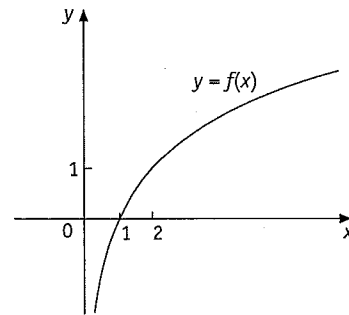
11 The diagram could be a sketch of the graph of:

(A) $y = 2^x$

(B) $y = 2^{-x}$

(C) $y = \log_2 x$

(D) $y = 2\ln x$

12 $\log_2 7 = ?$

(A) $\dfrac{\ln 7}{\ln 2}$ (B) $\dfrac{\ln 2}{\ln 7}$ (C) $2\ln 7$ (D) $7\ln 2$

SECTION II
Show all necessary working

88 marks

13 Simplify: **1 mark each**

  a $8^{x+1} \times 2^{5x} \div 4^{2-x}$ **b** $\log_6 45 + \log_6 20 - \log_6 25$

14 Find x if: **1 mark each**

  a $x^8 = 1\,679\,616$ **b** $(1-x)^3 = 0.512$ **c** $\log_x 16 = 4$

15 Find, correct to three decimal places: **1 mark each**

  a $\log_{10} 2.9$ **b** $\sqrt[5]{9.3187}$ **c** $\log_2 11$

16 If $\log_a 3 = 0.565$ and $\log_a 2 = 0.356$ find:

<div style="text-align: right">**1 mark each**</div>

   a $\log_a 6$ **b** $\log_a 9$ **c** $\log_a 1.5$

_____   _____   _____

_____   _____   _____

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_____   _____   _____

17 Find the value of x, correct to three decimal places, if:

<div style="text-align: right">**2 marks each**</div>

   a $5^x = 424$ **b** $1 - 3^x = 0.57$ **c** $6e^{2x+1} = 192$

18 Write down the exact value of:

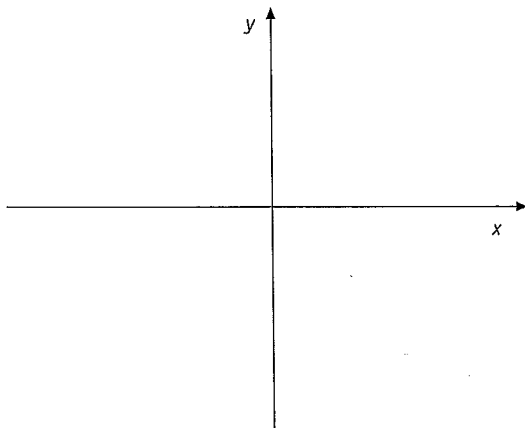<div style="text-align: right">**1 mark each**</div>

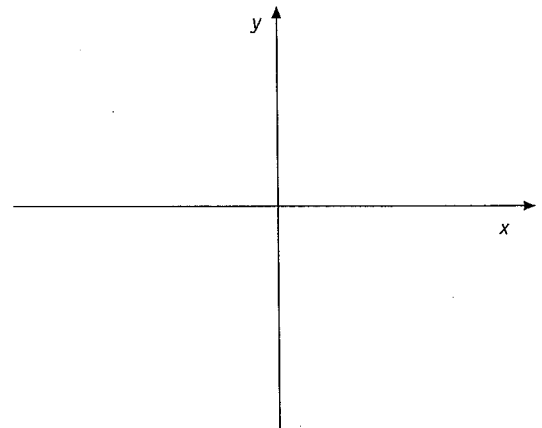   **a** $9 \ln e$ **b** $\ln e^4$ **c** $e^{\ln 8}$

19 Sketch the graph of:

<div style="text-align: right">**2 marks each**</div>

   a $y = 8^x$ **b** $y = \log_8 x$

20 Differentiate: **1 mark each**

a $y = 7^x$ **b** $y = e^x$ **c** $y = \ln x$

d $y = 3e^{-2x}$ **e** $y = \ln (5x - 4)$ **f** $y = \ln (x^2 + 6x)$

g $y = 5e^{7x-4}$ **h** $y = 4 \log_e(6 - 3x)$ **i** $y = e^{x^2}$

21 Find the derivative of: **3 marks each**

a $y = x^3 e^{2x}$ **b** $y = 2x \log_e x$

c $\dfrac{e^{6x}}{6x - 1}$ **d** $\dfrac{\ln x}{4x + 1}$

22 Find: 2 marks each

a $\displaystyle\int \frac{3}{x}\,dx$

b $\displaystyle\int e^{8x}\,dx$

c $\displaystyle\int \frac{4x}{2x^2 - 3}\,dx$

d $\displaystyle\int \frac{e^{-2x}}{2}\,dx$

e $\displaystyle\int \frac{3}{2x + 1}\,dx$

f $\displaystyle\int \frac{1}{4}e^{5-3x}\,dx$

23 Find the exact value of: 3 marks each

a $\displaystyle\int_1^e \frac{1}{x}\,dx$

b $\displaystyle\int_0^1 \frac{e^x}{2}\,dx$

c $\displaystyle\int_1^5 \frac{dx}{e^x}$

d $\displaystyle\int_1^4 \frac{2x + 5}{x^2 + 5x}\,dx$

e $\displaystyle\int_1^3 \frac{4}{7 - 2x}\,dx$

f $\displaystyle\int_0^2 e^{3x-4}\,dx$

24 Find the equation of the tangent to the curve $y = 2e^{x+1}$ at the point where $x = 0$ **3 marks**
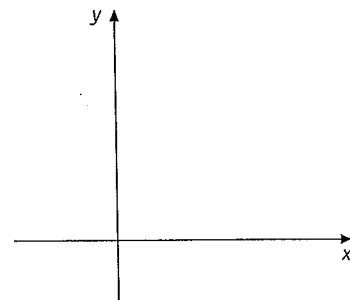
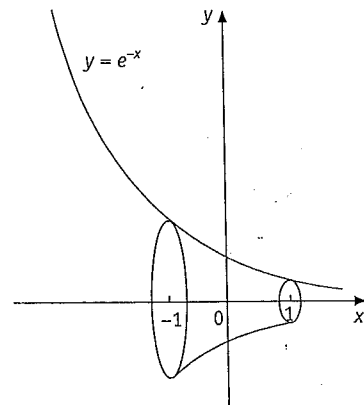**25** Find the coordinates of the stationary point of the curve $y = x \ln x$ **4 marks**

26 Find the area bounded by the curve $y = \dfrac{1}{x}$, the x-axis and the lines $x = 1$ and $x = 5$ **3 marks**

27 Find the volume of the solid of revolution formed when that portion of the curve $y = e^{-x}$ between $x = -1$ and $x = 1$ is rotated about the x-axis. **3 marks**

$y = e^{-x}$

Pages 114-119 1 A 2 D 3 D 4 C 5 A 6 B 7 A 8 B 9 D 10 C 11 C 12 A 13 a 2^{10x-1} b 2 14 a 6 b 0.2 c 2 15 a 0.462 b 1.563 c 3.459 16 a 0.921 b 1.13 c 0.209 17 a 3.759 b –0.768 c 1.233 18 a 9 b 4 c 8 19 a b (see next page) 20 a $(\ln 7)7^x$

b e^x **c** $\dfrac{1}{x}$ **d** $-6e^{-2x}$ **e** $\dfrac{5}{5x-4}$ **f** $\dfrac{2x+6}{x^2+6x}$ **g** $35e^{7x-4}$ **h** $\dfrac{-12}{6-3x}$ **i** $2xe^{x^2}$

21 a $x^2e^{2x}(2x+3)$ **b** $2(1+\log_e x)$ **c** $\dfrac{12e^{6x}(3x-1)}{(6x-1)^2}$ **d** $\dfrac{4x+1-4x\ln x}{x(4x+1)^2}$ **22 a** $3\ln x + C$

b $\dfrac{1}{8}e^{8x} + C$ **c** $\ln(2x^2-3) + C$ **d** $-\dfrac{1}{4}e^{-2x} + C$ **e** $\dfrac{3}{2}\ln(2x+1) + C$ **f** $-\dfrac{1}{12}e^{5-3x} + C$

23 a 1 **b** $\dfrac{e-1}{2}$ **c** $-\dfrac{1}{e^5} + \dfrac{1}{e}\left(\text{or}\ \dfrac{e^4-1}{e^5}\right)$ **d** $\ln 6$ **e** $2\ln 5$ **f** $\dfrac{1}{3e^4}(e^6-1)$ **24** $y = 2e(x+1)$ **25** $\left(\dfrac{1}{e}, -\dfrac{1}{e}\right)$ **26** $\ln 5$ units^2

27 $\dfrac{\pi}{2}\left(e^2 - \dfrac{1}{e^2}\right)$ units^3

19 a

19 b