LOGARITHMS & EXPONENTIALS ASSESSMENT TASK

Marks

(1) Given that $\log_x 5 = 1.32$ and $\log_x 6 = 1.78$, find : **6**

(a) $\log_x 25$ (b) $\log_x (1.2)$

(c) $\log_x 1$ (d) $\log_x 6x$

(2) (a) Evaluate $\log_3\left(\frac{1}{27}\right)$ **4**

(b) Calculate $\log_2 5$ correct to ***two*** decimal places.

(3) Sketch the graph, without calculus, the function $y = \log_e(x-1)$ and state the domain and range. **2**

(4) Differentiate : (a) e^{5x+2} (b) $\ln(x+1)$ **8**

(c) $(2x+1)e^{3x}$ (d) $\dfrac{\ln x}{e^x}$

(e) $\left[\log_e(2x)-3\right]^5$

(5) Find : (a) $\int e^{5-3x}\,dx$ (b) $\int \dfrac{1}{2x+1}\,dx$ **5**

(c) $\int_0^{\ln 2} \dfrac{e^x+1}{e^x}\,dx$

(6) Find the equation of the tangent to the curve $y = x\ln x$ at the point on the curve whose x–coordinate is 1. **3**

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(7) At any point on the curve $y = f(x)$, the gradient function is given by **3**

$$\frac{dy}{dx} = \frac{2x}{x^2+1}.$$

The point $(0, 2)$ lies on the curve. Find the equation of the curve.

(8) The region bounded by the curve $y = e^{2x}$, the line $x = 1$ and the coordinates axes is rotated through 360^0 about the x–axis. **5**

(a) Find the area of the region.

(b) Find the volume of the solid of revolution.

(9) Solve $\log_2(x+1) - \log_2(x-1) = 2$. **3**

(10) (a) Use the trapezoidal rule with 3 function values to approximate

$$\int_1^2 xe^{x^2}\,dx.$$

(b) Find the exact area of the above integral.

(c) Calculate the percentage error.

(11) A function is defined by the following :

$$f(x) = \begin{cases} e^{2x+1} & \text{for } x \geq 0 \\ e & \text{for } x < 0. \end{cases}$$

(a) Sketch the above function.

(b) Calculate the area under the curve and above the x–axis between $x = -2$ and $x = 2$.

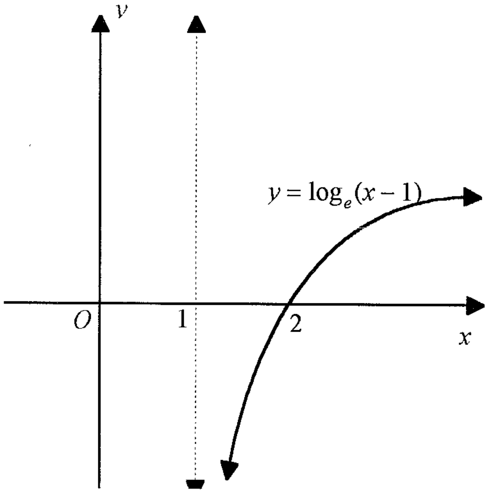
End of Assessment Task

Answers to Logs & exponentials task

(1) (a) 2.64 (b) 0.46 (c) 0 (d) 2.78

(2) (a) −3 (b) 2.32

(3)



$D: x > 1$ R: All real y

(4) (a) $5e^{5x+2}$ (b) $\frac{1}{x+1}$

(c) $e^{3x}(6x+5)$ (d) $\frac{\frac{1}{x} - \ln x}{e^x}$

(e) $\frac{5}{x}[\ln(2x) - 3]^4$

(5) (a) $-\frac{1}{3}e^{5-3x} + c$ (b) $\frac{1}{2}\ln(2x+1) + c$

(c) $\ln 2 + \frac{1}{2}$

(6) $x - y - 1 = 0$

(7) $f(x) = \ln(x^2 + 1) + 2$

(8) (a) $\frac{1}{2}(e^2 - 1)$ u^2 (b) $\frac{\pi}{4}(e^4 - 1)$ u^2

(9) $x = \frac{5}{3}$

(10) (a)

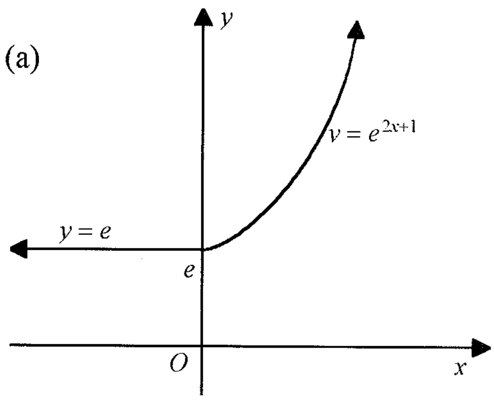
x	1	1.5	2
y	2.72	14.23	109.20

$A \approx 35.0945$ sq. units

(b) $\frac{e}{2}(e^3 - 1)$ u^2

(c) $\approx 35\%$

(11) (a)



(b) $\frac{e}{2}(3 + e^4)$.

ASSESS. TASK (LOGS/EXPON'S) 30/7/98.

(1) a) $\frac{d}{dx}\ln(5x-2) = \frac{5}{5x-2}$ using $\frac{d\ln U}{dU}\cdot\frac{dU}{dx}$

where $U = 5x-2$

b) $\frac{d\,e^{3-4x}}{dx} = -4\,e^{3-4x}$ using $\frac{d e^U}{dU}\cdot\frac{dU}{dx}$

where $U = 3-4x$.

c) $\frac{d}{dx}\left(\frac{e^{2x}}{4x}\right) = \frac{4x.(2e^{2x}) - e^{2x}.4}{(4x)^2}$ using $\frac{VU' - UV'}{V^2}$

$= \frac{4e^{2x}\{2x-1\}}{16x^2} = \frac{e^{2x}\{2x-1\}}{4x^2}$

d) $\frac{d(4+e^{7x})^3}{dx} = 3(4+e^{7x})^2.\,7e^{7x}$ using $\frac{dU^3}{dU}\cdot\frac{dU}{dx}$

where $U = (4+e^{7x})$

e) $\frac{d(x^2\ln x)}{dx} = x^2.\frac{1}{x} + \ln x.\,2x$

$= x(1+2\ln x)$

(2) a) $\int \frac{6}{2x-3}\,dx = 3.\int\frac{2}{2x-3}\,dx = 3\ln(2x-3)+C$

using $\int f'(x).\frac{1}{f(x)}\,dx = \ln f(x)+C$

b) (i) $\int_{-1}^{2} e^{2x+3}\,dx = \frac{1}{2}\left[e^{2x+3}\right]_{-1}^{2}$ using $\int f'(x)\,e^{f(x)}\,dx = e^{f(x)}+C$.

$= \frac{1}{2}\left[e^{7} - e\right]$

(ii) $\int_1^2 x - x^{\frac{1}{2}} - \frac{3}{x}\,dx = \left[\frac{x^2}{2} - \frac{2x^{3/2}}{3} - 3\ln x\right]_1^2$

$= \left[\frac{4}{2} - \frac{2.2\sqrt{2}}{3} - 3\ln 2\right] - \left[\frac{1}{2} - \frac{2}{3} - 0\right]$

$= 2 - \frac{4\sqrt{2}}{3} - 3\ln 2 + \frac{1}{6}$

$= \frac{13-8\sqrt{2}}{6} - 3\ln 2 = \frac{13-8\sqrt{2}-18\ln 2}{6}$

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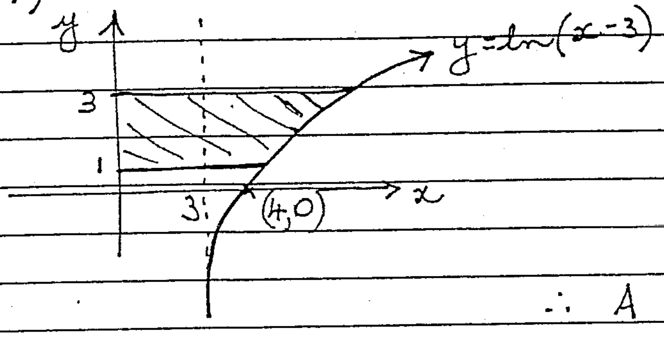
(3) a) $\log_a 12 = \log_a(4\times3) = \log_a 4 + \log_a 3 = 4\cdot3$

b) $\log_a 27 = \log_a(3^3) = 3\log_a 3 = 5\cdot82$

c) $\log_a\sqrt{3} = \log_a 3^{\frac{1}{2}} = \frac{1}{2}\log_a 3 = 0\cdot97$

d) $\log_a 0\cdot75 = \log_a\frac{3}{4} = \log_a 3 - \log_a 4 = \cancel{-1\cdot58}\ -0\cdot41$

(4)



$$\text{Area} = \int_1^3 x\,dy \quad \text{where } y = \log_e(x-3)$$

i.e. $e^y = x-3$

So, $x = 3+e^y$

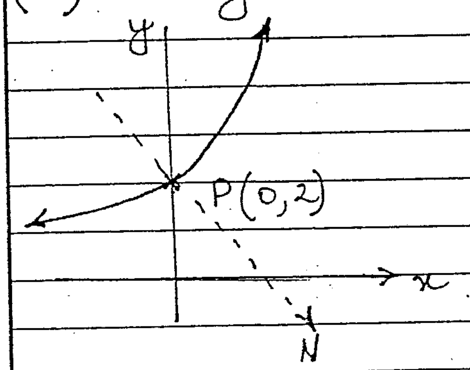
$$\therefore A = \int_1^3 (3+e^y)\,dy$$

$$= \Big[3y+e^y\Big]_1^3$$

$$= (9+e^3)-(3+e)$$

$$= (6+e^3-e) \quad u^2$$

(5) $y = 2e^{3x}$.



Curve cuts y-axis $(x=0)$ at $(0,2)$

Now $\frac{dy}{dx} = 2(3e^{3x})$

i.e. $\frac{dy}{dx} = 6e^{3x}$

When $\underline{x=0}$, grad. of tang^t, $\frac{dy}{dx} = 6.e^0 = 6$

$\therefore$ Using $m_N \times m_T = -1$,

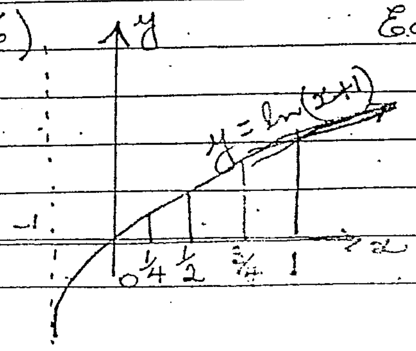
$m_N = -\frac{1}{6}$ is grad. of normal

Eqn of normal is $y-2 = -\frac{1}{6}(x-0)$

" " " $6y-12 = -x$

" " " $x+6y-12 = 0$ (OR $y = -\frac{1}{6}x+2$)

(6)



Each strip is $\frac{1}{4}$ unit wide $(b-a) = \frac{1}{4}$ for each strip.

$$A \approx \frac{\frac{1}{4}}{6}\left\{f(0)+f(\tfrac{1}{4})+4f(\tfrac{1}{8})\right\} + \frac{\frac{1}{4}}{6}\left\{f(\tfrac{1}{4})+f(\tfrac{1}{2})+4f(\tfrac{3}{8})\right\}$$

$$+ \frac{\frac{1}{4}}{6}\left\{f(\tfrac{1}{2})+f(\tfrac{3}{4})+4f(\tfrac{5}{8})\right\} + \frac{\frac{1}{4}}{6}\left\{f(\tfrac{3}{4})+f(1)+4f(\tfrac{7}{8})\right\}$$

$$\approx \frac{1}{24}\left[f(0)+f(1)+2\left\{f(\tfrac{1}{4})+f(\tfrac{3}{4})+f(\tfrac{1}{2})\right\}+4\left\{f(\tfrac{1}{8})+f(\tfrac{3}{8})+f(\tfrac{5}{8})+f(\tfrac{7}{8})\right\}\right]$$

(6) (Cont'd)

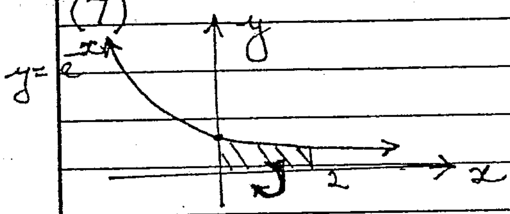
i.e. $A \approx \frac{1}{24}\left[\ln 1 + \ln 2 + 2\left\{\ln \frac{5}{4} + \ln \frac{7}{4} + \ln \frac{6}{4}\right\} + 4\left\{\ln \frac{9}{8} + \ln \frac{11}{8} + \ln \frac{13}{8} + \ln \frac{15}{8}\right\}\right]$

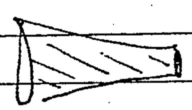
$\approx \frac{1}{24}\left[\ln 2 + 2 \ln \frac{210}{64} + 4 \ln \frac{19305}{4096}\right]$

using $\ln a + \ln b = \ln ab$, etc

$\approx 0.38 \quad u^2$

(7)





$V = \pi \int_0^2 y^2 \, dx$

$= \pi \int_0^2 (e^{-x})^2 \, dx$

$= \pi \int_0^2 e^{-2x} \, dx$

$= \frac{-\pi}{2}\left[e^{-2x}\right]_0^2 \quad u^3$.

$= \frac{-\pi}{2}\left[e^{-4} - e^0\right] \quad u^3$

$= \frac{-\pi}{2}\left[e^{-4} - 1\right] \quad u^3$.

(8) $y = x \ln x$ — (1)

$\frac{dy}{dx} = x.\frac{1}{x} + \ln x . 1$

$= 1 + \ln x$ — (2)

$\frac{d^2y}{dx^2} = \frac{1}{x}$ — (3)

From (2), T.P. occur for $\boxed{\frac{dy}{dx} = 0}$

i.e. $1 + \ln x = 0$

$\ln x = -1$

ie $x = e^{-1} = \frac{1}{e} \Rightarrow y = \frac{1}{e}\ln\left(\frac{1}{e}\right) = \frac{1}{e}\ln(e^{-1}) = \frac{-1}{e}\ln e$

$= -\frac{1}{e}$

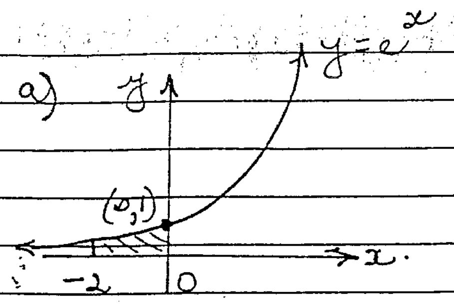
If $x = \frac{1}{e}$, (3) $\Rightarrow \frac{d^2y}{dx^2} = \frac{1}{\left(\frac{1}{e}\right)} = e > 0$

since $\ln e = 1$

i.e. At $x = \frac{1}{e}$, there is a MIN. PT.

$\left(\frac{1}{e}, -\frac{1}{e}\right)$ is a MIN. PT.

(9) a)

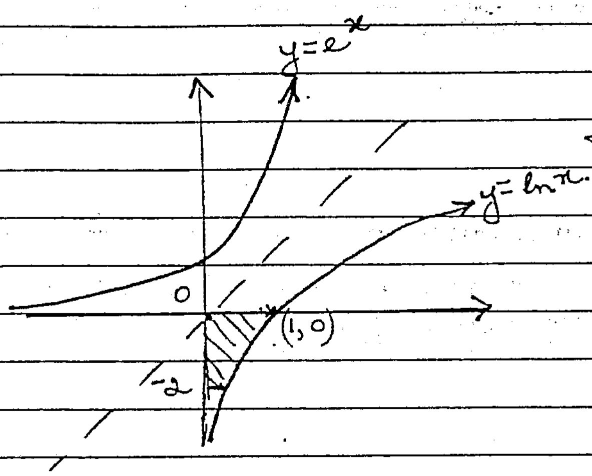


$$A = \int_{-2}^{0} e^x \, dx$$

$$= \left[e^x \right]_{-2}^{0} \quad u^2$$

$$= \left[e^0 - e^{-2} \right] \quad u^2$$

$$= \left[1 - \frac{1}{e^2} \right] \quad u^2$$



The sh. area here is identical ~~to~~ that above because $y = \ln x$ is the reflection of $y = e^x$ in the line $y = x$.