

Methods of integration

TOPIC TEST

Time allowed: 1 hour

Total marks = 100

SECTION I Multiple-choice questions

8 marks

Instructions • This section consists of 8 multiple-choice questions

- Each question is worth 1 mark
- Fill in only ONE CIRCLE
- Calculators may be used

1 $\int x^4 dx = ?$

- (A) $x^5 + C$ (B) $\frac{1}{5}x^5 + C$ (C) $5x^5 + C$ (D) none of these

2 $[x^2 - 3x]_{-1}^2 = ?$

- (A) -6 (B) 0 (C) 2 (D) 6

3 $\int_0^2 dx = ?$

- (A) 0 (B) 1 (C) 2 (D) none of these

4 Which is the correct formula for Simpson's rule? $\int_a^b f(x) dx \approx$

- (A) $\frac{b-a}{2} [f(a) + f(b)]$ (B) $\frac{b-a}{n} \left[f(a) + f\left(\frac{a+b}{2}\right) + f(b) \right]$
(C) $\frac{b-a}{n} \left[f(a) + 6f\left(\frac{a+b}{2}\right) + f(b) \right]$ (D) $\frac{b-a}{6} \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right]$

5 If $\int_0^4 f(x) dx = [x^3]_0^4$ then $f(x) = ?$

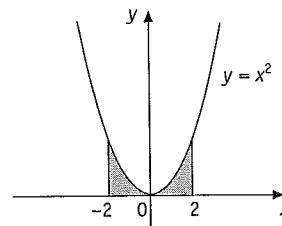
- (A) $\frac{x^2}{3}$ (B) $3x^2$ (C) $\frac{x^4}{3}$ (D) $3x^4$

6 Which is correct?

The required area could be found by evaluating:

I $\int_{-2}^2 x^2 dx$ II $2 \int_0^2 x^2 dx$

- (A) I only (B) II only
(C) either I or II (D) neither I nor II

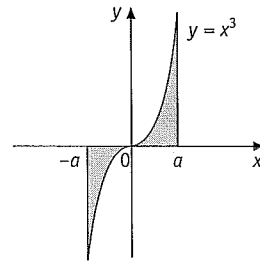


7 The volume obtained by rotating that section of the curve $y = f(x)$ between $x = a$ and $x = b$ about the x -axis is given by:

- (A) $V = \int_a^b \pi x dx$ (B) $V = \int_a^b \pi x^2 dy$ (C) $V = \int_a^b \pi y dx$ (D) $V = \int_a^b \pi y^2 dx$

8 $\int_{-a}^a x^3 dx = ?$

- (A) 0 (B) $\frac{a^4}{4}$
 (C) $\frac{a^4}{2}$ (D) not enough information



SECTION II

92 marks

Show all necessary working

9 Find the value of:

3 marks each

a $\int_0^3 6x^2 dx$

b $\int_{-1}^1 (4x - 7) dx$

c $\int_1^2 3x^{-4} dx$

d $\int_1^3 (4x - 1)^3 dx$

10 Find these indefinite integrals:

3 marks each

a $\int 12x^2 dx$

b $\int (x^2 - 8x + 5) dx$

c $\int (5x - 2)^8 dx$

d $\int \frac{2}{x^3} dx$

e $\int \sqrt{6x - 1} dx$

f $\int \frac{8}{\sqrt{2x + 3}} dx$

11 Use one application of the trapezoidal rule to approximate $\int_1^3 \frac{1}{2x} dx$

5 marks

12 Use the trapezoidal rule to approximate $\int_1^3 2^x dx$ using 5 function values.

6 marks

13 Use Simpson's rule, with three function values, to estimate $\int_0^2 \frac{4}{3^x} dx$

5 marks

14 Use the table of values and Simpson's rule to find an approximate value of $\int_1^2 f(x) dx$

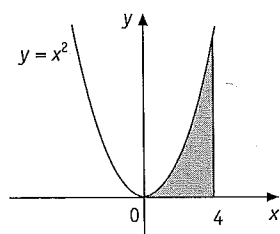
x	1	1.25	1.5	1.75	2
$f(x)$	0.20	0.13	0.09	0.06	0.04

6 marks

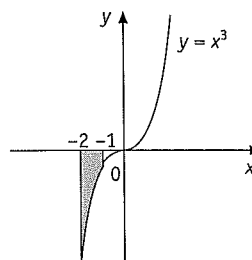
15 Find the shaded area:

5 marks each

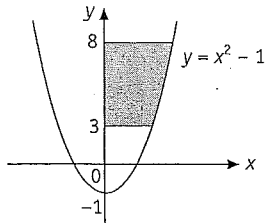
a



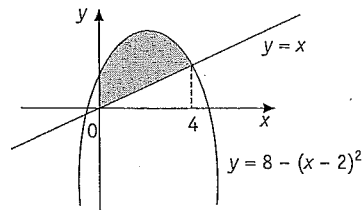
b



15 c



d

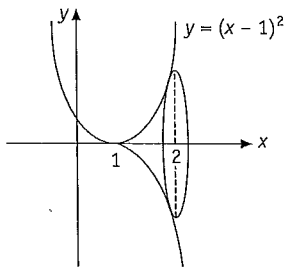


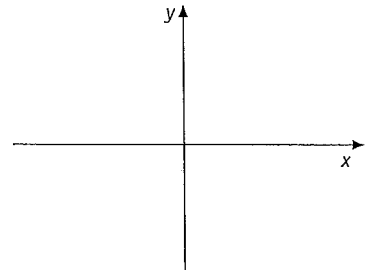
16 Find the volume of the solid formed when the given section of the curve is rotated about the x-axis:

a $y = (x - 1)^2$ between $x = 1$ and $x = 2$

b $y = x^2 + 1$ between $x = 0$ and $x = 2$

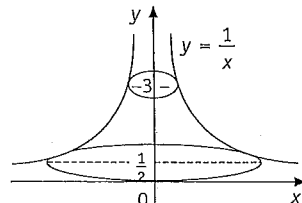
6 marks each





17 Find the volume of the solid of revolution formed when the positive branch of the hyperbola $y = \frac{1}{x}$ between $y = \frac{1}{2}$ and $y = 3$ is rotated about the y-axis.

8 marks



Pages 55-59 1 B 2 A 3 C 4 D 5 B 6 C 7 D 8 A 9 a 54 b -14 c $\frac{7}{8}$ d 910 10 a $4x^3 + C$ b $\frac{x^3}{3} - 4x^2 + 5x + C$

c $\frac{(5x - 2)^9}{45} + C$ d $-\frac{1}{x^2} + C$ e $\frac{\sqrt{(6x - 1)^3}}{9} + C$ f $8\sqrt{2x + 3} + C$ 11 $\frac{2}{3}$ 12 8.7 [1 d.p.] 13 $3\frac{7}{27}$ 14 0.1 [1 d.p.]

15 a $21\frac{1}{3}$ units² b 3.75 units² c $12\frac{2}{3}$ units² d $18\frac{2}{3}$ units² 16 a $\frac{\pi}{5}$ units³ b $\frac{206\pi}{15}$ units³ 17 $\frac{5\pi}{3}$ units³