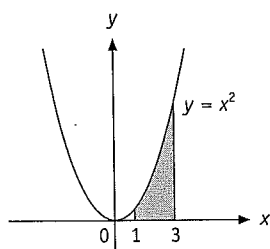# Methods of integration

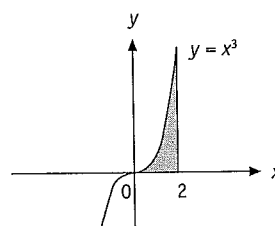## Area (1)

QUESTION **1**   Complete:

If $f(x) \geq 0$ for $a \leq x \leq b$, then $\displaystyle\int_a^b f(x)\,dx$ gives the area beneath the _____ above the _____ and between the lines $x =$ _____ and $x =$ _____.
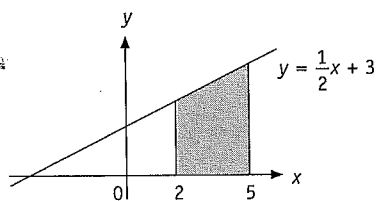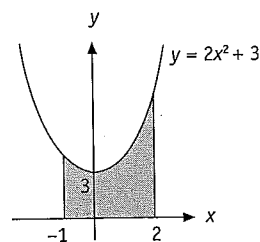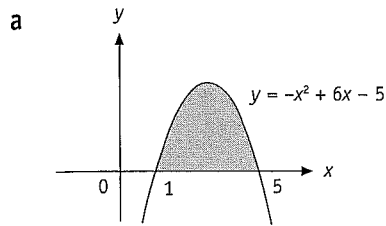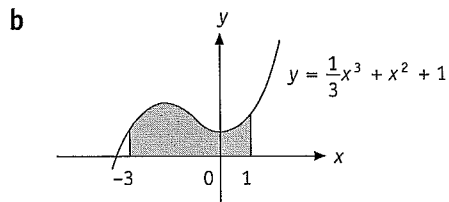
QUESTION **2**   Find the shaded area:

a

b

c

d

Methods of integration

Area (2)

QUESTION **1**   Find the shaded area:

a

$y = -x^2 + 6x - 5$

b

$y = \frac{1}{3}x^3 + x^2 + 1$

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QUESTION **2**   Find the area bounded by the curve $y = (x - 1)^2$, the x-axis and the lines $x = 2$ and $x = 3$

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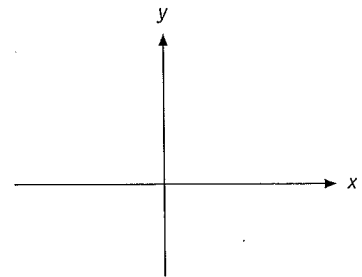
QUESTION **3**   Find the area beneath the curve $y = 4 - x^2$ and above the x-axis.
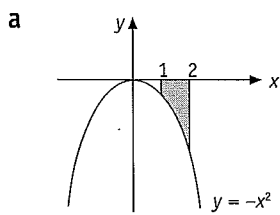
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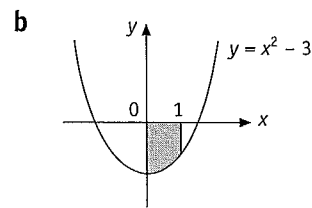
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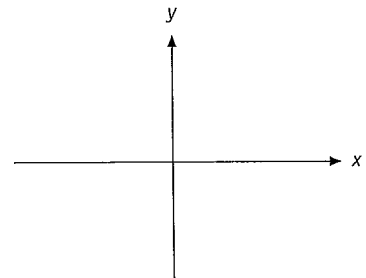
Methods of integration

Areas below the x-axis (1)

QUESTION **1** Complete:

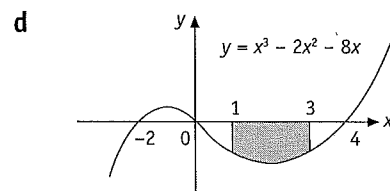If $f(x) < 0$ for $a \le x \le b$, then $\displaystyle\int_a^b f(x)dx$ will be _____ (positive/negative). The area bounded by the curve the x-axis and the lines $x = a$ and $x = b$ is equal to the _____ _____ of the integral.
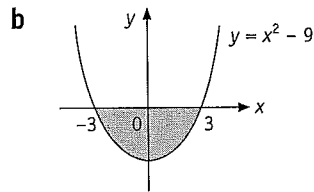
QUESTION **2** Find the shaded area:

a

b

$y = x^2 - 3$

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c

$y = x^2 - 2x$

d

$y = x^3 - 2x^2 - 8x$

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Methods of integration
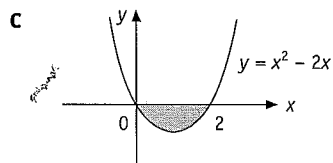
Areas below the x-axis (2)

QUESTION **1**   Find the shaded area. (Use symmetry to simplify the working.)

a

$y = x^3$

b

$y = x^2 - 9$

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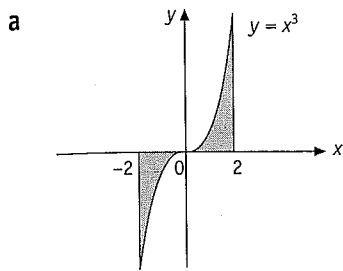
QUESTION **2**

a The point closest to the y-axis where the curve $y = -x^2 + 8x - 12$ cuts the x-axis is where $x = p$. Find the value of p.

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$y = -x^2 + 8x - 12$

b Find the shaded area.

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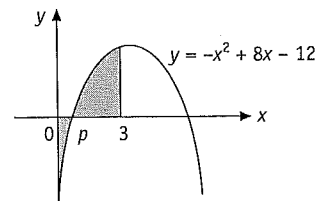
Methods of integration

Areas bounded by the *y*-axis (1)

QUESTION **1**

a Make x the subject of the equation $y = x^2 + 1$

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b Find the area shaded in the diagram.

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QUESTION **2**   Find the shaded area:

a

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b

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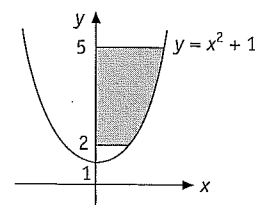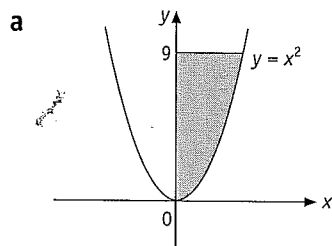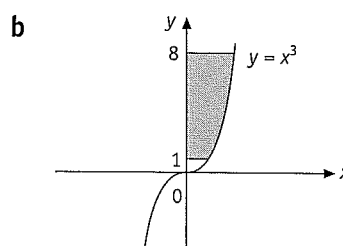
Methods of integration

Areas bounded by the y-axis (2)

QUESTION **1**   Find the shaded area:

a

b

_____

_____

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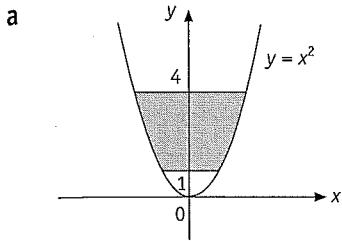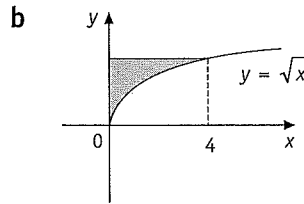
QUESTION **2**

a   Find the area bounded by the curve $y = x^4$, the x-axis, and the ordinates $x = 0$ and $x = 2$.

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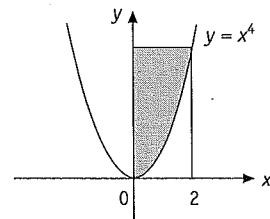
b   Hence find the shaded area.

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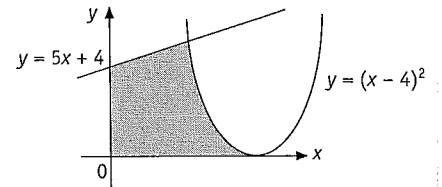# Methods of integration

Combined areas (1)

QUESTION **1**

a Find the x-coordinates of the points of intersection of the line $y = 5x + 4$ and the parabola $y = (x - 4)^2$

$y = 5x + 4$

$y = (x - 4)^2$

b Find the shaded area.

QUESTION **2**

a Solve:

i $x^2 - 13x + 30 = 0$

ii $2x^2 = x^2 - 13x + 30$

b Find the shaded area:

$y = 2x^2$

$y = x^2 - 13x + 30$

Methods of integration
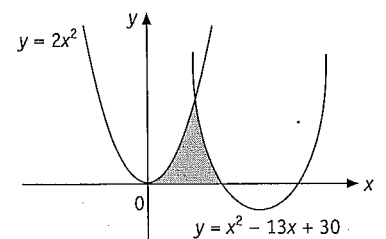
Combined areas (2)

QUESTION **1**   Find the shaded area:

a

$y = x^2 + 3$ $y = 2x + 3$

b

$y = x^2$ $y = 8 - x^2$

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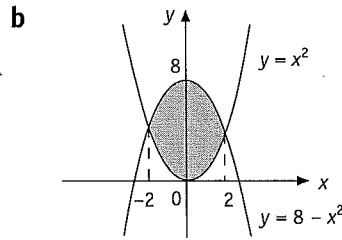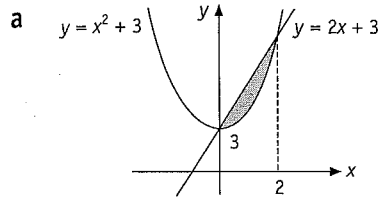
QUESTION **2**   The diagram shows the graphs of $y = 4x^2 - 8x + 12$ and $y = x^2 - 2x + 21$

a   Find the x-coordinates of the points of intersection of the curves.

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_____

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$y = x^2 - 2x + 21$
$y = 4x^2 - 8x + 12$
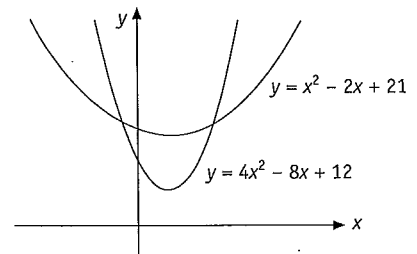
b   Find the area between the curves.

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Page 44 1 curve, *x*-axis, *a*, *b* 2 a $8\frac{2}{3}$units² b 4 units² c $14\frac{1}{4}$units² d 15 units²

Page 45 1 a $10\frac{2}{3}$ units² b $6\frac{2}{3}$ units² 2 $2\frac{1}{3}$ units² 3 $10\frac{2}{3}$units²

Page 46 1 negative, absolute value 2 a $2\frac{1}{3}$ units² b $2\frac{2}{3}$units² c $1\frac{1}{3}$units² d $29\frac{1}{3}$ units²

Page 47 1 a 8 units² b 36 units² 2 a $p = 2$ b $12\frac{1}{3}$units²

Page 48 1 a $x = \pm\sqrt{y - 1}$ b $4\frac{2}{3}$ units² 2 a 18 units² b $11\frac{1}{4}$units²

Page 49 1 a $9\frac{1}{3}$ units² b $2\frac{2}{3}$units² 2 a 6.4 units² b 25.6 units²

Page 50 1 a $x = 1$ and $x = 12$ b 15.5 units² 2 a i $x = 3$ or $x = 10$ ii $x = -15$ or $x = 2$ b $9\frac{1}{6}$ units²

Page 51 1 a $1\frac{1}{3}$units² b $21\frac{1}{3}$ units² 2 a $x = -1$ and $x = 3$ b 32 units²