OUR LADY OF THE SACRED HEART COLLEGE
KENSINGTON

STUDENT – NAME / NUMBER _____

MATHEMATICS TEACHER _____

Year 11

Mathematics

Assessment 3

2008

Time allowed: **45 minutes**

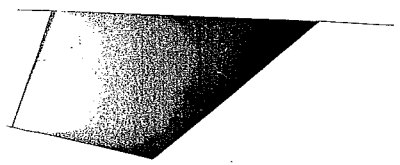
Assessed Outcomes

P3: Performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and equations

P4: chooses and applies appropriate arithmetic & algebraic techniques

Directions to Candidates

- Attempt all questions
- START EACH QUESTION ON A NEW PAGE
- Show all necessary working on the paper
- Calculators may be used
- Good Luck!!



Question One: (12 marks)

(a) Complete the following statements:

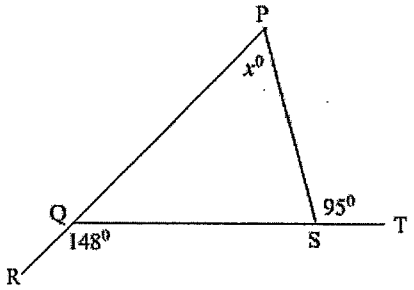
(i) The sum of the angles at a point is 1

(ii) The angles of a square are ..... and of an equilateral triangle are 2

(iii) The corresponding intercepts cut off on two transversals by three parallel lines are 1

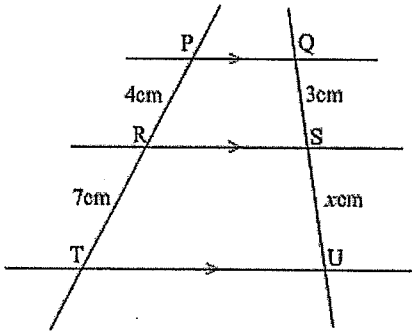
(b) Find x in each of the following diagrams, giving brief reasons.

(i)



2

(ii)



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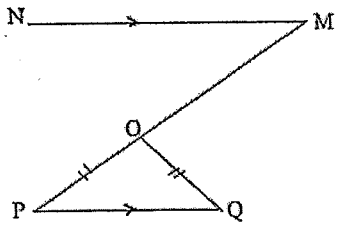
Question One continued.

(c)



In the diagram NM ∥ PQ and OP = OQ. Prove that $\angle NMP = \angle OQP$ **4**

Question Two: (12 marks)

(a) (i) Plot on a number plane the point A (3 , 4). Draw the interval AO where O is the origin **1**

(ii) Plot the points B (3 , 0) and C (0 , -4) on your diagram. **1**

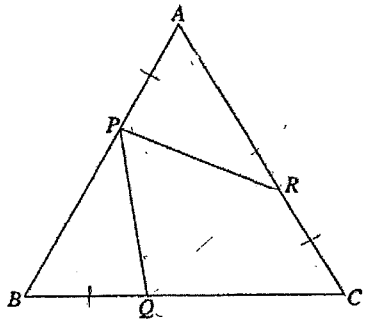
(iii) Show that the line BC has equation $3y = 4x - 12$. **2**

(iv) Show that OABC is a parallelogram. Give reasons. **1**

(v) Find the area of the parallelogram OABC. **1**

(vi) Calculate the length of the diagonal AC. **2**

(b)



Δ ABC is equilateral. $AP = BQ = CR$.

(i) Prove that triangles APR and BQP are congruent. **2**

(ii) Prove that $\angle QPR = 60°$ **1**

(iii) Prove that Δ PQR is equilateral **1**

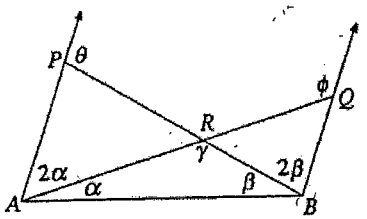
Question Three: (12 marks)

(a) A is the point (-1, 3), and a line BC has the equation $2x-5y=12$.

(i) Find the equation of the line through A perpendicular to BC. **3**

(ii) If this line meets BC at D, find the coordinates of D. **3**

(b)



AP and BQ are parallel.

(i) Prove that $\alpha+\beta=60^0$ **2**

(ii) Find the value of γ. **2**

(iii) Find the value of $\theta+\phi$. **2**

Question 1

a) i) 180° x 360° 0

ii) 90°, 60° 2

iii) ~~equal~~ proportional 1

b) $\angle PQS = 180 - 148$ (straight line)

$\angle PQS = 32°$

$\angle PSQ = 180 - 95$ (straight $\angle$)

$= 85°$

$\therefore \angle x = 180 - 85 - 32$ ($\angle$ sum of Δ)

$= 63°$ ✓ 2

(i) $\frac{4}{7} = \frac{3}{x}$ (corresponding intercepts cut off 2 transversals are proportional)

$4x = 21$

$x = \frac{21}{4}$

$x = 5\frac{1}{4}$

$x = 5.25$ cm ✓ 2

c) $\angle NMP = \angle OPQ$ (alternate $\angle$'s, ~~NM~~ NM ∥ PQ) ✓

$\therefore \angle OQP = \angle OPQ$ (base $\angle$'s of isosceles Δ) ✓

$\therefore$ ~~$\angle$~~ $\angle OQP = \angle NMP$ ✓ 4

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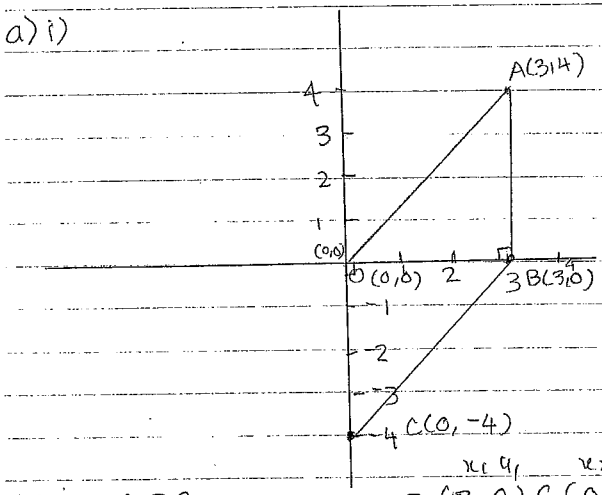
Question 2

Physics
MS Green

a) i)



2

$x_1\ y_1$ $x_2\ y_2$

B (3, 0) C (0, -4)

ii) Eqn of BC

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{y_2 - y_1}{x_2 - x_1}(x - 3)$

$y = \frac{-4 - 0}{0 - 3}(x - 3)$

$y = \frac{-4}{-3}(x - 3)$

$y = \frac{4}{3}(x - 3)$

$y = \frac{4}{3}x - \frac{12}{3}$

$3y = 4x - 12$ 2

(iii)

(iv) ✓

Question 2 cont??

Alyssa

b) i) AP = BQ (given)

∠PAR = ∠PBQ = 60° (∠s of equilateral △)

∴ AR = PB (Why?)

∴ △APR ≡ △BQP

ii) ∠QPR = 60 (opp ∠'s of ∥-ogram)

↳ How do you know?

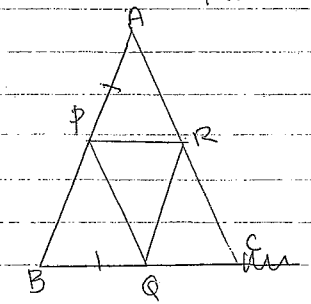
construct QR

iii) ∠QRP = 60° (proven)

∴ ∠PQR and ∠QRP = $\frac{180-60}{2}$ (Why?)

∴ ∠PQR and ∠QRP = 60°

∴ △PQR is equilateral



△APR: x + y = 120 (∠ sum of △)

∠APR + ∠RPQ + ∠BPQ = 180 (St ∠ APB)

x + ∠QPR + y = 180°

∠QPR + x + y = 180°

∠QPR = 180 − 120

∠QPR = 60°

iii) As △'s APR

Question 2 cont??

iv) D of OA = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $x_1\ y_1$ $x_2\ y_2$ O(0,0) A(3,4)

$= \sqrt{(3-0)^2+(4-0)^2}$

$= \sqrt{(3)^2+(4)^2}$

$= \sqrt{25}$

$= 5$

D of AB = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $x_1\ y_1$ $x_2\ y_2$ A(3,4) B(3,0)

$= \sqrt{(3-3)^2+(0-4)^2}$

$= \sqrt{(-4)^2}$

$= \sqrt{16}$

$= 4$

D of BC = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $x_1\ y_1$ $x_2\ y_2$ B(3,0) C(0,−4)

$= \sqrt{(0-3)^2+(-4-0)^2}$

$= \sqrt{(-3)^2+(-4)^2}$

$= \sqrt{25}$

$= 5$

D of OC = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $x_1\ y_1$ $x_2\ y_2$ O(0,0) C(0,−4)

$= \sqrt{(0-0)^2+(-4-0)^2}$

$= \sqrt{(-4)^2}$

$= \sqrt{16}$

$= 4$

∴ BC = OA (same distance, equal length)

AB = OC (same distance, equal)

∴ OABC is a parallelogram (opp sides are equal)

v) D of CA = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $x_1\ y_1$ C(0,−4) $x_2\ y_2$ A(3,4)

$= \sqrt{(3-0)^2+(4+4)^2}$

$= \sqrt{(3)^2+(8)^2}$

$= \sqrt{73}$ (vi)

$A = \frac{1}{2} \times \sqrt{73} \times 3$

$= \frac{\sqrt{73} \times 3}{2}$ A = b × h

$= 4 \times 3$

$= 12$

$= \frac{3\sqrt{73}}{2}$ units²

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Question 3

a) i) A $(\overset{x_1}{-1}, \overset{y_1}{3})$ BC $\parallel$ to $2x-5y=12$

$m = 2/5$

$\therefore$ m of A $= -2$ NO!

$y - y_1 = m(x - x_1)$

$y - 3 = -2(x+1)$ $\quad m_1 \times m_2 = -1$

$y - 3 = -2x - 2$

$y = -2x - 2 + 3$

$y = -2x + 1 \Rightarrow 2x + y = 1$ 2

ii) $2x - 5y = 12 \quad -$ I

$2x + y = -1 \quad +$ II

$-6y = 11$

$y = \frac{11}{6}$ ✓

sub y into I $\Rightarrow 2x - 5(\frac{11}{6}) = 12$

$2x - \frac{55}{6} = 12$

$2x = 12 + \frac{55}{6}$

$x = \frac{72}{6} + \frac{55}{6}$

$x = \frac{127}{6}$ 3

b) i) $\angle AQB = 2\alpha$ (alt $\angle$'s, AP $\parallel$ BQ)

$\angle PAQ = \angle RQB$ (" " " " ")

$\therefore 2\alpha + \alpha + 2\beta + \beta = 180^\circ$ ($\angle$ sum of $\triangle$ ABQ)

$3\alpha + 3\beta = 180 \quad \therefore \alpha + \beta = 60^\circ$

$3(\alpha+\beta) = 180$

$\alpha + \beta = \frac{180}{3}$ 2

ii) $\alpha + \beta = 60^\circ$ (proven)

$\therefore \gamma = 180 - 60$ ($\angle$ sum of $\triangle$) ARB

$\therefore \gamma = 120^\circ$ ✓ 2

Question 3 cont

Alyssa

iii) $\theta = 180 - 2\beta$ (corresponding $\angle$s)

$\phi = 180 - 2\alpha$ (corresponding $\angle$s)

$180 = (180 - 2\beta) + (180 - 2\alpha)$

$= 180 + 2\beta + 180 - 2\alpha$

$180 = 360 - 2\beta - 2\alpha$

$2\beta + 2\alpha = 180$

$180 - 2\alpha + 2\alpha = 180$

$(180 - 2\beta) + (180 - 2\alpha)$

$= 180 - 2\beta + 180 - 2\alpha$

$\theta + \phi = 360 - 2\alpha - 2\beta$

$180 - 2\beta + 180 - 2\alpha$

$\theta + \phi = 360 - 2\alpha - 2\beta$

$\theta + \phi = 360 - 2(\alpha + \beta)$

$\alpha + \beta = 60$

$\therefore \theta + \phi = 360 - 2(60)$

$= 360 - 120$

$= 240^\circ$ ✓

$\phi = \alpha + 3\beta$ (ext $\angle$ of $\triangle$ ABQ = sum of opp interior)

$\theta = \beta + 3\alpha$ (ext $\angle$ of $\triangle$ ARD P = sum of opp int).

$\phi = \theta = \alpha + \beta + 3\alpha + 3\beta$

$= 60 + 180$

$= 240^\circ$ 2