206. A function $y = f(x)$ has zero concavity for all real numbers x. What can be said of its graph?

207. The plumbing in Mustafa's house runs along his outside wall. During summer the sun heats the water in the pipes and as a result when he turns on the **cold** tap, the water that comes out is initially quite hot, cooling down to the temperature of the main water supply over a period of a few seconds. The temperature of the water T (in degrees C) at time t (in seconds) is given by

$$T = \frac{60t + 80}{4t + 1}; \quad t \geq 0.$$

(a) What is the initial temperature of the water?

(b) Find a formula for the rate of change of temperature with respect to time.

(c) What is the rate of change of the temperature after 8 seconds?

(d) When is the water cooling at a rate of $1°C$ per second?

(e) By considering an appropriate limit determine the temperature of the main water supply.

(f) To what limit does the rate of change of temperature tend?

208. At time t (in months) during the growth of a certain tree its root volume (in cubic metres) is given by $R = \dfrac{15t}{t+1}$ and its leaf volume (in cubic metres) is given by $L = \dfrac{40t^2}{(t+1)^2}$.

(a) Show that the tree has more roots than leaves after half a month but that the situation has been reversed after a full month.

(b) When is the root volume equal to the leaf volume?

(c) By considering appropriate limits determine the eventual size of both the root and leaf systems.

(d) At what rate is the root volume growing after one month?

(e) (*) At which point in time are the two systems growing at the same rate?

209. (**) Arteriosclerosis is the process by which plaque forms on the inner surface of arterial walls, blocking the flow of blood and leading to gangrene, heart attack, stroke, death and other more serious consequences. Suppose that the percentage A of the artery which is blocked at age t (in years) is given by

$$A = \frac{100\beta t}{100 + \beta t} \quad t \geq 0.$$

where $\beta = 1, 2, 3, 4, 5$ is a lifestyle parameter.

Emmy who doesn't drink or smoke and exercises regularly has a value of $\beta = 2$, while Amir who drinks, smokes and gets little exercise has a value of $\beta = 5$.

(a) Find a formula for the percentage of arterial blockage as a function of age for both Emmy and Amir.

(b) Verify that both of them are born with clear arteries and will (eventually) have 100% of the artery blocked.

(c) For each individual determine the age at which the arteries will be 60% blocked.

(d) For each individual determine the **rate** at which the percentage blockage is growing on their 40th birthday.

(e) At which age will their percentage plaque be growing in both individuals at the same rate?

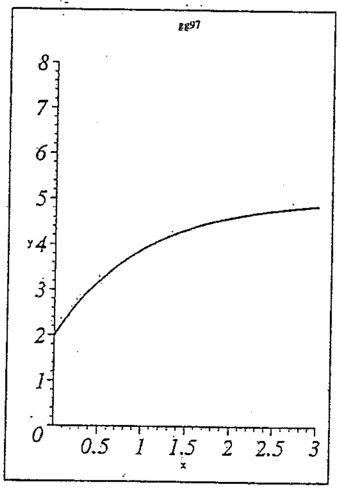
SOLUTIONS

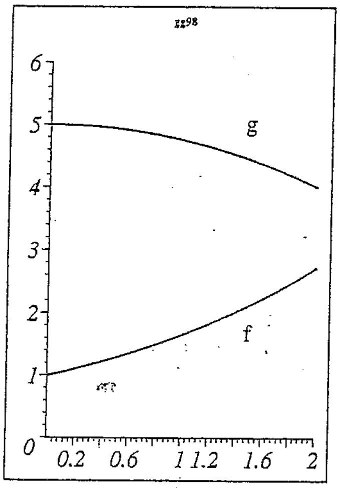
Product, quotient and chain rules

201. (a) $\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}}$ (b) $\frac{-7}{(3x-2)^2}$ (c) $10x(x^2+1)^4$
(d) $\frac{3}{2\sqrt{3x-1}}$ (e) $\frac{-x}{2\sqrt{1-x}} + \sqrt{1-x}$ (f) $26x^3(x^2-1)^{12} + 2x(x^2-1)^{13}$

202. $-\frac{1}{4}x^{-\frac{3}{2}} - \frac{3}{4}x^{-\frac{5}{2}}$, $80x^2(x^2+1)^3 + 10(x^2+1)^4$

203. For I) $x_1 : +,+,-$, $x_2 : +,-,-$, $x_3 : -,0,+$, $x_4 : +,+,-$.
For II) $x_1 : +,+,0$, $x_2 : +,0,-$, $x_3 : +,-,0$.
For III) $x_1 : -,+,0$, $x_2 : +,+,0$.

204. 

205. 

206. It is linear.

207. (a) 80 ° C (b) $\frac{dT}{dt} = \frac{-260}{(4t+1)^2}$ (c) $-0.24°$ /sec (d) After 3.78 seconds.
(e) 15° (f) 0

208. (a) $t = \frac{1}{2}$: $R = 5$, $L \approx 4.44$ $t = 1$: $R = 7.5$, $L = 10$.
(b) After 3/5 of a month. (c) $R \to 15$ m ^{3} , $L \to 40$ m ^{3} .
(d) 15/4 m ^{3} /month (e) After 3/13 months.

209. (a) $A = \frac{100t}{t+50}$ for Emmy, $A = \frac{100t}{t+20}$ for Amir.
(b) Proof (Hint: Consider $\lim_{t\to\infty} A$).
(c) Amir: 30 years, Emmy: 75 years.
(d) Amir: 5/9 % per year, Emmy: 50/81 % per year.
(e) 31.62 years.