CHAPTER 6
Series and applications

Introduction

QUESTION **1** Fill in the missing term in each series:

a $3 + 5 + 7 + \_\_\_\_\_ + 11 + 13 + \ldots$

b $1 - 3 + 9 - 27 + \_\_\_\_\_ - 243 + \ldots$

c $87 + 68 + \_\_\_\_\_ + 30 + 11 - 8 - \ldots$

d $8 + 12 + 18 + \_\_\_\_\_ + 40.5 + 60.75 + \ldots$

e $1 + 2 + 5 + 14 + 41 + 122 + \_\_\_\_\_ + \ldots$

f $3 + 7 + 15 + 31 + \_\_\_\_\_ + 127 + \ldots$

g $\_\_\_\_\_ + 3750 + 1500 + 600 + 240 + \ldots$

h $56 + \_\_\_\_\_ + 22 + 5 - 12 - 29 - \ldots$

QUESTION **2** Evaluate:

a $\sum_{k=1}^{3} (k + 2)$

b $\sum_{k=1}^{6} (5k + 3)$

c $\sum_{r=1}^{5} r^2$

d $\sum_{n=2}^{5} (7n - 4)$

e $\sum_{k=1}^{6} 3(2)^{k-1}$

f $\sum_{k=1}^{4} 9000(0.1)^{k-1}$

g $\sum_{k=3}^{6} 2^k$

h $\sum_{k=1}^{3} 625(-0.6)^{k-1}$

QUESTION **3** Write using sigma notation:

a $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15$

b $1 + 2 + 4 + 8 + 16 + 32 + 64$

Series and applications

Arithmetic series

QUESTION **1** Complete:
In an arithmetic series each term differs from the previous term by a fixed amount called the common _____.

QUESTION **2** The given series is an arithmetic series. True or false?

a $3 + 9 + 15 + 21 + 27 + \ldots$ _____

b $2 + 4 + 8 + 16 + 32 + \ldots$ _____

c $56 + 52 + 48 + 44 + 40 + \ldots$ _____

d $18 + 21 - 24 + 27 - 30 + \ldots$ _____

e $512 + 256 + 128 + 64 + 32 + \ldots$ _____

f $4 - 2 - 8 - 14 - 20 - \ldots$ _____

g $1 + 4 + 9 + 16 + 25 + 36 + \ldots$ _____

h $1 + 2 + 4 + 7 + 11 + 16 + \ldots$ _____

QUESTION **3** Write down the first six terms of the series with first term a and common difference d:

a $a = 3, d = 7$ _____

b $a = 2, d = 5$ _____

c $a = -8, d = 4$ _____

d $a = 6, d = -2$ _____

e $a = -5, d = -3$ _____

f $a = -23, d = 17$ _____

QUESTION **4** Find the common difference for each series:

a $1 + 6 + 11 + 16 + 21 + \ldots$ _____

b $9 + 17 + 25 + 33 + 41 + \ldots$ _____

c $32 + 60 + 88 + 116 + 144 + \ldots$ _____

d $18 + 15 + 12 + 9 + 6 + \ldots$ _____

e $-11 - 15 - 19 - 23 - 27 - \ldots$ _____

f $-17 - 10 - 3 + 4 + 11 + \ldots$ _____

QUESTION **5** Write down the next three terms of each arithmetic series.

a $312 + 369 + 426 + \ldots$ _____

b $82 + 65 + 48 + \ldots$ _____

c $-34 - 21 - 8 + \ldots$ _____

d $101 + 37 - 27 - \ldots$ _____

The n^{th} term of an arithmetic series (1)

QUESTION **1** Find the given term of an arithmetic series with first term a and common difference d:

a $a = 6$, $d = 5$, 6^{th} term

b $a = 2$, $d = 8$, 7^{th} term

c $a = 10$, $d = 3$, 9^{th} term

d $a = 7$, $d = -3$, 12^{th} term

e $a = -12$, $d = 2$, 5^{th} term

f $a = -5$, $d = -4$, 10^{th} term

QUESTION **2** Find the given term of the series:

a $5 + 18 + 31 + 44 + \ldots$ (17^{th} term)

b $29 + 23 + 17 + 11 + \ldots$ (19^{th} term)

QUESTION **3** Find the first term and common difference of the arithmetic series with n^{th} term, T_n

a $T_n = 2n + 7$

b $T_n = 7n - 2$

c $T_n = 14 - 3n$

QUESTION **4** Find an expression, in simplest form, for the n^{th} term of the arithmetic series with:

a $a = 2$, $d = 3$

b $a = 7$, $d = 2$

c $a = 4$, $d = -2$

Series and applications

The n^{th} term of an arithmetic series (2)

QUESTION **1** Find an expression for the n^{th} term of the series: $4 + 12 + 20 + 28 + 36 + \ldots$

QUESTION **2** Find the number of terms in the series $2 + 19 + 36 + 53 + \ldots + 444$

QUESTION **3** The 7^{th} term of an arithmetic series is 19 and the 10^{th} term is 40. Find the fifth term.

QUESTION **4** The n^{th} term of an arithmetic series is given by $T_n = 12 + 11n$. Find:

a the 9^{th} term

b which term is equal to 155

c whether 234 is a term of the series

d the first term which is greater than 1000

Sum to *n* terms of an arithmetic series (1)

QUESTION **1** Find the sum of the first n terms of the arithmetic series with first term a and common difference d:

a $a = 6, d = 7, n = 8$

b $a = 5, d = 11, n = 4$

c $a = -3, d = 1, n = 15$

d $a = 56, d = -12, n = 7$

e $a = -2, d = -5, n = 16$

f $a = -32, d = 19, n = 40$

QUESTION **2** Find the sum of the first n terms of the series:

a $13 + 17 + 21 + 25 + \ldots \ (n = 12)$

b $128 + 92 + 56 + 20 + \ldots \ (n = 14)$

QUESTION **3** Find the sum to n terms of the arithmetic series with first term a and last term l:

a $a = 2, l = 48, n = 10$

b $a = 95, l = 2093, n = 38$

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Sum to *n* terms of an arithmetic series (2)

QUESTION **1** Find an expression for the sum to n terms of the arithmetic series with first term a and common difference d:

a $a = 34,\ d = 18$

b $a = 7,\ d = -12$

QUESTION **2** Find an expression for the sum to n terms of the series:

a $56 + 97 + 138 + 179 + \ldots$

b $85 + 78 + 71 + 64 + \ldots$

QUESTION **3** For the series $129 + 145 + 161 + 177 + \ldots + 801$ find:

a the number of terms

b the sum of the series

Sum to *n* terms of an arithmetic series (3)

QUESTION **1** Find the sum of the series:

a $37 + 50 + 63 + 76 + \ldots + 310$

b $726 + 681 + 636 + 591 + \ldots + 6$

QUESTION **2** The sum of the first n terms of a series is given by $S_n = 2n^2 + 7n$. Find:

a the sum of the first eight terms **b** the sum of the first seven terms **c** the eighth term

QUESTION **3** The sum of the first n terms of a series is given by $S_n = 4n^2 + 3n$. Find an expression for the n^{th} term.

Page 141 1 a 9 b 81 c 49 d 27 e 365 f 63 g 9375 h 39 2 a 12 b 123 c 55 d 82 e 189 f 9999 g 120 h 475

3 a $\sum_{k=1}^{8}(2k-1)$ or $\sum_{k=0}^{7}(2k+1)$ b $\sum_{k=1}^{7}2^{k-1}$ or $\sum_{k=0}^{6}2^{k}$

Page 142 1 difference 2 a true b false c true d false e false f true g false h false 3 a 3 + 10 + 17 + 24 + 31 + 38 b 2 + 7 + 12 + 17 + 22 + 27 c −8 − 4 + 0 + 4 + 8 + 12 d 6 + 4 + 2 + 0 − 2 − 4 e −5 − 8 − 11 − 14 − 17 − 20 f −23 − 6 + 11 + 28 + 45 + 62 4 a 5 b 8 c 28 d −3 e −4 f 7 5 a 483 + 540 + 597 b 31 + 14 − 3 c 5 + 18 + 31 d −91 − 155 − 219

Page 143 1 a 31 b 50 c 34 d −26 e −4 f −41 2 a 213 b −79 3 a 9, 2 b 5, 7 c 11, −3 4 a $3n - 1$ b $2n + 5$ c $6 - 2n$

Page 144 1 $8n - 4$ 2 27 3 5 4 a 111 b 13th term c not a term d $T_{90} = 1002$

Page 145 1 a 244 b 86 c 60 d 140 e −632 f 13 540 2 a 420 b −1484 3 a 250 b 41 572

Page 146 1 a $9n^2 + 25n$ b $13n - 6n^2$ 2 a $\frac{n}{2}(41n + 71)$ b $\frac{n}{2}(177 - 7n)$ 3 a 43 b 19 995

Page 147 1 a 3817 b 6222 2 a 184 b 147 c 37 3 $8n - 1$