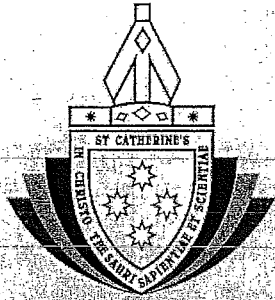


Student Number: _____



St. Catherine's School
Waverley

2007

HIGHER SCHOOL CERTIFICATE
ASSESSMENT TASK 3 – 20%
CLASS TEST: 4th June

Mathematics

General Instructions

- Working time – 55 minutes
- Start each question on a new page in your answer booklet.
- If any additional booklet is used, please label it clearly and attach it to the appropriate booklet.
- Write using black or blue pen only.
- Board-approved calculators may be used.
- All necessary working must be shown.
- Marks may be deducted for careless or badly arranged work.

Total marks – 43

- Attempt Questions 1–3
- Questions are not of equal value

TEACHER'S USE ONLY

Question 1	/13
Question 2	/15
Question 3	/15
TOTAL	/43

Question 1

Marks

a) Find $\frac{dy}{dx}$ for

(i) $y = e^{3x+5}$

1

(ii) $y = xe^x$

2

b) Evaluate $\int_0^2 e^{2x} dx$ correct to 1 decimal place

2

c) Find the gradient of the tangent to $f(x) = \frac{e^{-x}}{x}$ at $x=1$

3

d) Find $\frac{d}{dx} e^{x^2}$ and hence find the exact value of $\int_0^1 xe^{x^2} dx$

4

Question 2

Marks

a) Find $\frac{dy}{dx}$ for

(i) $y = x^2 \log_e x$

2

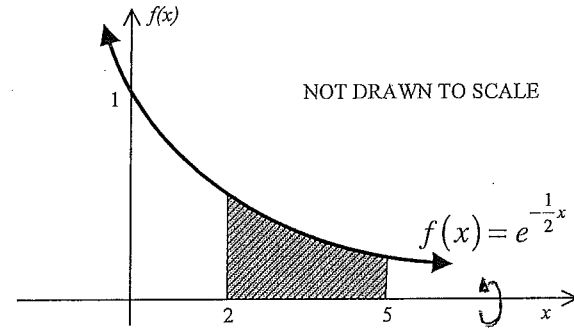
(ii) $y = \log_e \sqrt{3x-5}$

3

b) Evaluate $\int_e^{e^2} \frac{6}{x} dx$ as an exact value

3

e)



4

The section of the curve $f(x) = e^{-\frac{1}{2}x}$ between $x = 2$ and $x = 5$ shown above is rotated about the x -axis. Find the volume enclosed.

d) Find the equation of the tangent to the curve $y = 4 + 3 \log_e x$ at the point where $x = 1$

3

Question 3

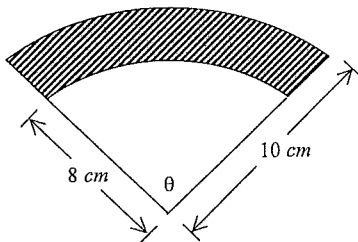
Marks

a) What is the exact value of $\sin \frac{2\pi}{3}$

1

b) Find the area shaded in the diagram below if $\theta = 2$

3



c) (i) Sketch the curve $y = 2\sin 2x$ for $0 \leq x \leq 2\pi$

3

showing a clear scale on both axes.

(ii) By graphing the line $y = 1$ on your diagram, show that the equation

$$2\sin 2x = 1$$

has four solutions for $0 \leq x \leq 2\pi$

1

d) Differentiate:

(i) $y = e^{2 \tan x}$

2

(ii) $y = \sin^3 x \cos x$

3

(iii) $y = \log_e(\cos x)$

2

End of Paper

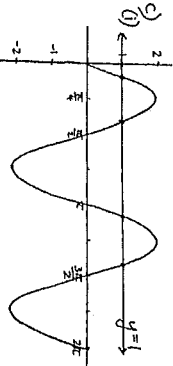
2007 MATHEMATICS ASSESSMENT TASK 3
SOLUTIONS

Qn	Solutions	Marks	Comments+Criteria
1.	<p>A) (i) $y = e^{3x+5}$ $y' = f'(x)e^{f(x)}$ $= 3e^{3x+5}$</p> <p>(ii) $y = xe^x$ $u=x \quad v=e^x$ $u'=1 \quad v'=e^x$ $y' = vu' + uv'$ $= e^x + xe^x$ $= e^x(1+x)$</p>	1 1 1	for correct answer for correct use of product rule for correct answer
	<p>B) $\int_0^2 e^{2x} dx = \frac{1}{2} \int_0^2 2e^{2x} dx$ $= \frac{1}{2} [e^{2x}]_0^2$ $= \frac{1}{2} [e^4 - e^0]$ $= \frac{e^4}{2} - \frac{1}{2}$ $= 26.8 \text{ (to 1 d.p.)}$</p>	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	for re-writing integral for correct integration for substitution for answer to 1 d.p.
	<p>c) $f(x) = \frac{e^{-x}}{x} = (xe^x)^{-1}$ $f'(x) = -(xe^x)^{-2} \times e^x(1+x)$ $= \frac{-e^x(1+x)}{x^2 e^{2x}}$ $= \frac{-(1+x)}{x^2 e^x}$ $\therefore \text{when } x=1, m_T = \frac{-2}{e}$</p>	1 1 1	for correct use of chain or quotient rule for correct simplification

Qn	Solutions	Marks	Comments+Criteria
1	<p>d) $\frac{d}{dx} e^{x^2} = 2xe^{x^2}$ $\therefore e^{x^2} = \int 2xe^{x^2} dx$ $e^{x^2} = 2 \int xe^{x^2} dx$ $\Rightarrow \frac{e^{x^2}}{2} = \int xe^{x^2} dx$ $\therefore \int_0^1 xe^{x^2} dx = \left[\frac{e^{x^2}}{2} \right]_0^1$ $= \frac{e^1}{2} - \frac{e^0}{2}$ $= \frac{1}{2}(e-1)$</p>	1 1 1	for differentiation for appropriate use of derivative to form integral for integration for solution after substitution
2	<p>A) (i) $y = x^2 \log_e x$ $u=x^2 \quad v=\ln x$ $u'=2x \quad v'=\frac{1}{x}$ $y' = vu' + uv'$ $= 2x \ln x + x^2 \times \frac{1}{x}$ $= 2x \ln x + x$ $= x(2 \ln x + 1)$</p> <p>(ii) $y = \log \sqrt{3x-5}$ $y = \log (3x-5)^{\frac{1}{2}}$ $y = \frac{1}{2} \log (3x-5)$ $y' = \frac{1}{2} \times \frac{3}{(3x-5)}$ $= \frac{3}{6x-10}$</p>	1 1 1 $\frac{1}{2}$	for correct use of product rule for correct answer for use of log law for correct answer

Qn	Solutions	Marks	Comments+Criteria
3)	<p>(i) $y = \underbrace{\sin^3 x}_u \underbrace{\cos x}_v$</p> <p>$u = \sin^3 x$ $v = \cos x$ $\left(\frac{1}{2}\right)$</p> <p>$u' = 3\cos x \sin^2 x$ $v' = -\sin x$ $\left(\frac{1}{2}\right)$</p> <p>$y' = vu' + uv'$</p> <p>$= 3\cos^2 x \sin^2 x + \sin^4 x$</p> <p>$= \sin^2 x (3\cos^2 x - \sin^2 x)$</p> <p>* using identities to simplify</p> <p>$y' = \sin^2 x (3(1-\sin^2 x) - \sin^2 x)$ $y' = \sin^2 x (3\cos^2 x - (1-\cos^2 x))$</p> <p>$= \sin^2 x (3 - 4\sin^2 x)$ $= \sin^2 x (4\cos^2 x - 1)$</p>	1	
	<p>(ii) $y = \log_e (\cos x)$</p> <p>$y = \ln (\cos x)$</p> <p>$y' = \frac{-\sin x}{\cos x}$</p> <p>$y' = -\tan x$</p>	1 1/2	for correct derivative of $\cos x$
		1/2	for simplifying to $-\tan x$.

Qn	Solutions	Marks	Comments+Criteria
2	<p>b) $\int_e^{e^2} \frac{6}{x} dx = 6 \int_e^{e^2} \frac{1}{x} dx$</p> <p>$= 6 [\ln x]_e^{e^2}$</p> <p>$= 6 (\ln e^2 - \ln e)$</p> <p>$= 6 (2\ln e - \ln e)$</p> <p>$= 6 (2-1)$</p> <p>$= 6$</p>	1	for integration
	<p>c) $y = e^{-\frac{1}{2}x}$</p> <p>$\therefore y^2 = (e^{-\frac{1}{2}x})^2 = e^{-x}$</p> <p>$\therefore V = \pi \int_2^5 e^{-x} dx$</p> <p>$= -\pi \int_2^5 e^{-x} dx$</p> <p>$= -\pi [e^{-x}]_2^5$</p> <p>$= -\pi (e^{-5} - e^{-2})$</p>	1 1/2	for correct integration
	<p>d) $y = 4 + 3 \log_e x$</p> <p>$\frac{dy}{dx} = \frac{3}{x}$</p> <p>when $x=1$, $y = 4 + 3 \ln 1$</p> <p>$= 4$</p> <p>$x=1, m_1 r = \frac{3}{1} = 3$</p> <p>$\therefore y - y_1 = m(x - x_1)$</p> <p>$y - 4 = 3(x - 1)$</p> <p>$y - 4 = 3x - 3$</p> <p>$3x - y + 1 = 0$</p>	1 1/2	for derivative
		1/2	for correct y coord.
		1/2	for correct $m_1 r$
		1	for correct equation

Qn	Solutions	Marks	Comments+Criteria
3	<p>4) $\sin \frac{2\pi}{3} = \sin (\pi - \frac{2\pi}{3})$</p> <p>$= \sin \frac{\pi}{3}$</p> <p>$= \frac{\sqrt{3}}{2}$</p>	1	for exact value
	<p>b) Area = $\frac{1}{2} (R^2 \theta - r^2 \theta)$</p> <p>$= \frac{1}{2} (10^2 \times 2 - 8^2 \times 2)$</p> <p>$= \frac{1}{2} (200 - 128)$</p> <p>$= \frac{1}{2} \times 72$</p> <p>$= 36 \text{ cm}^2$</p>	2	for correct answer
	<p>c) </p> <p>(i) See above</p> <p>The line $y=1$ has 4 points of intersection with $y=2\sin 2x$ over $0 \leq x \leq 2\pi$</p> <p>\therefore there are 4 solutions</p>	1 1/2	for correct graph of $y=1$
		1/2	for showing 4 points of intersection/solutions
		1	for correct amplitude
		1	for correct derivative of $\tan x$
		1	for correct answer