SYDNEY GRAMMAR.

5G 2008 **FUNCTIONS and GRAPHS (2F2G)** 4th February

Attempt all questions. Show all working. Use your own paper.

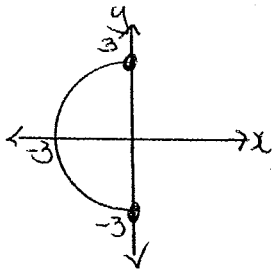
1. If $f(x) = x^2 + 5x$, find:
   (a) $f(-3)$
   (b) k where $f(k) = 14$
   (c) $f(a+h) - f(a)$

2. A function is given by:
$$F(x) = \begin{cases} 1-x, & \text{for } x \leq 0 \\ -x^2, & \text{for } x > 0 \end{cases}$$
   Sketch the graph of $F(x)$.

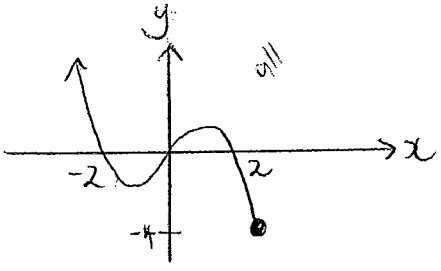
3. Sketch the following curves on separate number planes. Show all important features.
   (a) $y = x^2 - 36$
   (b) $x^2 + y^2 = 64$
   (c) $x - 2y = 4$
   (d) $y = -x^2 + 6x + 40$
   (e) $y = -\sqrt{16 - x^2}$
   (f) $xy = 4$

4. Determine which of the following relations are functions. Write down the domain and range of each function.

a)



b)



5. If $f(x) = \dfrac{1}{x-2}$,
   (a) Write down the domain of $f(x)$.
   (b) Find any vertical or horizontal asymptotes. Show your reasoning.
   (c) Plot any relevant points.
   (d) Sketch the graph of $f(x)$.
   (e) Write down the range of $f(x)$.

5G

1. $f(x) = x^2 + 5x$

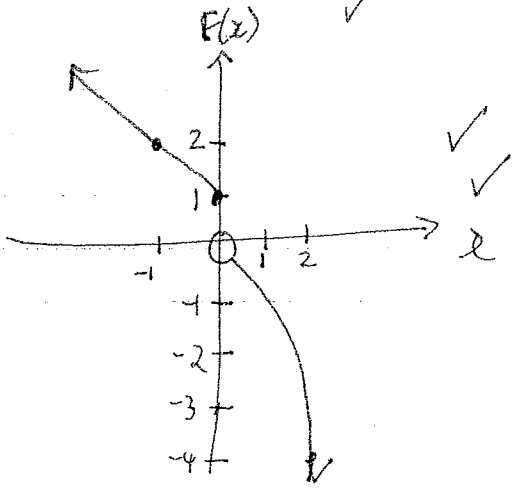
a) $f(-3) = (-3)^2 + 5 \times -3$
$= 9 - 15$
$= -6$ ✓

b) $f(k) = 14$
$14 = k^2 + 5k$
$k^2 + 5k - 14 = 0$
$(k-2)(k+7) = 0$

$k = 2$ or -7 ✓ ✓

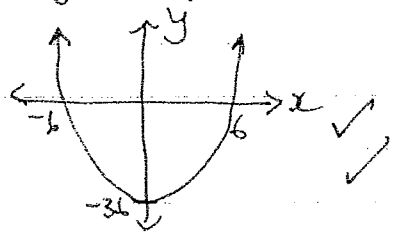
c) $f(a+h) - f(a)$
$= (a+h)^2 + 5(a+h) - (a^2 + 5a)$ ✓
$= a^2 + 2ha + h^2 + 5a + 5h - a^2 - 5a$
$= 2ah + h^2 + 5h$ ✓

2.



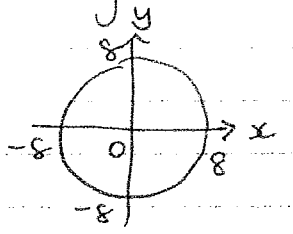
✓ ✓

3. a) $y = x^2 - 36$
$= (x-6)(x+6)$



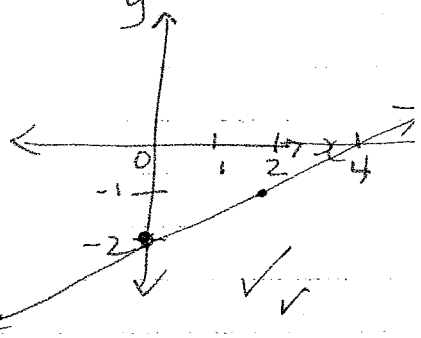
✓ ✓

b) $x^2 + y^2 = 64$
$x^2 + y^2 = 8^2$



✓ ✓
centre (0,0)
radius = 8

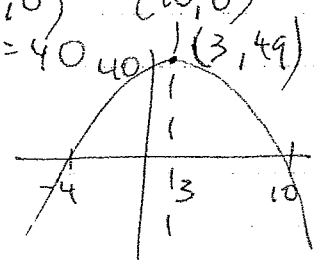
c) $x - 2y = 4$
$x - 4 = 2y$
$y = \frac{x}{2} - 2$



✓ ✓

d) $y = -x^2 + 6x + 40$
$= -(x^2 - 6x - 40)$
$= -(x+4)(x-10)$

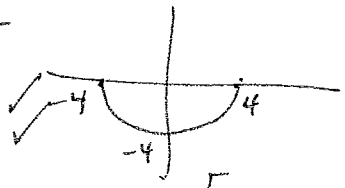
✓ $y = 0$ $x = -4$ $x = 10$
$(-4, 0)$ $(10, 0)$
✓ $x = 0$ $y = 40$



✓ vertex
$x = \frac{-b}{2a}$
$= \frac{-6}{-2}$
$x = 3$

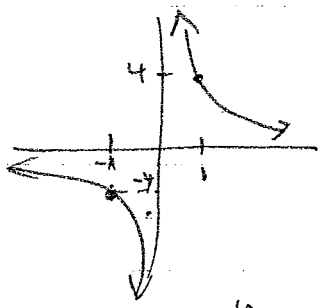
$y = -(3+4)(3-10)$
$= -(7)(-7) = 49$

e) $y = -\sqrt{16 - x^2}$



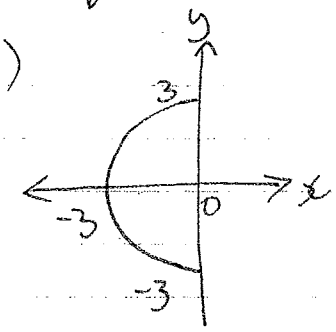
f) $xy = 4$

$y = \frac{4}{x}$



✓ ✓

4. a)



relation as it is not a function as two points have the same x-coordinate. ✓

domain $-3 \leq x \leq 0$ ✓

range $-3 \leq y \leq 3$ ✓

b) function ✓

domain $x \leq 2$ ✓

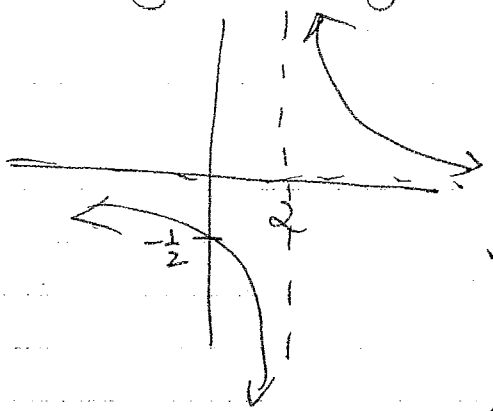
range $y \geq -4$ ✓

5. $f(x) = \frac{1}{x-2}$.

a) Domain $x \neq 2$ ✓

e) range $y \neq 0$ ✓

d)



✓

c) $x = 0$ $y = -\frac{1}{2}$ ✓ $(0, -\frac{1}{2})$

b) Asymptotes $x = 2$ ✓

$y = 0$ ✓

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$\sqrt{4} + 6$