SYDNEY GRAMMAR

5G 2009 | **TRIGONOMETRY 1** | 6th March

Use your own paper. Attempt all questions.

1. (a) What are the two acute angles of a right angled triangle whose sides are 3 cm, 4 cm and 5 cm (nearest degree)?

   (b) In the triangle XYZ, $\angle X = 30°$, $\angle Y = 82°$ and $XY = 460\,\text{mm}$. Find YZ (nearest millimetre).

   (c) In the triangle JKL, $\angle J = 27°$, $JK = 23\,\text{cm}$ and $KL = 15\,\text{cm}$. Find the values of $\angle L$ (nearest degree).

2. Simplify without using a calculator:

   (a) $\cos 45°$,

   (b) $\cot 150°$,

   (c) $\dfrac{\tan 60° - \tan 45°}{1 + \tan 60° \tan 45°}$. Express your answer with a rational denominator.

3. If $\sin\theta = \dfrac{2}{5}$ find all possible values for $\cot\theta$.

4. Sketch $y = \sin\theta$ for $-180° \le \theta \le 180°$.

5. Simplify $\dfrac{\sin^2\theta}{1 - \sin^2\theta}$.

6. Prove the identity $\tan A \sin A + \cos A = \sec A$.

7. Solve the following equations:

   (a) $\sin x = 0$ for $-180° \le x \le 180°$,

   (b) $\sin x = \frac{1}{2}$ for $0° \le x \le 360°$,

   (c) $\cos^2 2x = \frac{1}{2}$ for $0° \le x \le 360°$,

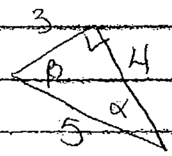
   (d) $3\sec^2\theta + 2\tan\theta - 4 = 0$ for $0° \le \theta \le 360°$(nearest degree).

8. Two adjacent sides of a parallelogram have lengths of 12 cm and 15 cm and the included angle is 50°. Find the area of the parallelogram correct to the nearest square centimetre.

9. A plane flies from Sydney Airport for 70 nautical miles on a bearing of 142° and then on another course for 60 nautical miles. If it is now due east of its starting point, find the bearing of its second course, to the nearest degree.

10. If θ is acute and $\sin\theta = \frac{1}{\sqrt{3}}$ show that $\dfrac{\tan\theta}{1 - \sec\theta} = -\sqrt{2} - \sqrt{3}$ and find the value of this fraction when θ is obtuse.

5G

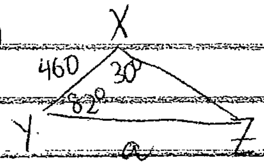
1a) 

$\sin\alpha = \frac{3}{5}$

$\alpha \doteqdot 37$ ✓

$\beta = 90 - \alpha$

$= 53$ ✓ (angle sum of $\triangle$)

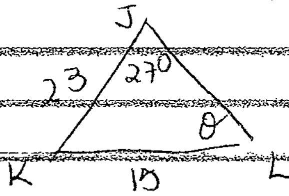
b) 

$\angle XZY = 68$ ✓

$\frac{a}{\sin 30^\circ} = \frac{460}{\sin 68^\circ}$ ✓

$a = \frac{460 \sin 30^\circ}{\sin 68^\circ}$ ✓

$= 248$ (nearest mm)

c) 

$\frac{23}{\sin\theta} = \frac{15}{\sin 27}$ ✓

$\frac{\sin\theta}{23} = \frac{\sin 27}{15}$

$\sin\theta = \frac{23 \sin 27}{15}$ ✓

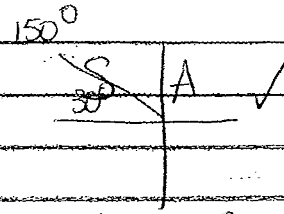
$\theta \doteqdot 44$

ASS check ambigious case

$180 - \theta = 136$ ✓

valid

2. a) $\cos 45^\circ = \frac{1}{\sqrt{2}}$

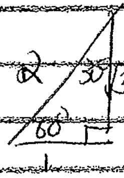
b) $\cot 150^\circ$ 

✓

$= \frac{1}{\tan 150^\circ}$

$= \frac{1}{-\frac{1}{\sqrt{3}}}$

$\tan 30^\circ = \frac{1}{\sqrt{3}}$ ✓

$= -\sqrt{3}$ ✓ 

c) $\frac{\tan 60^\circ - \tan 45^\circ}{1 + \tan 60^\circ \tan 45^\circ}$

$= \frac{(\sqrt{3}-1)}{(1+\sqrt{3})}$ ✓ $\times \frac{(1-\sqrt{3})}{(1-\sqrt{3})}$

$= \frac{(\sqrt{3}-1)(1-\sqrt{3})}{1-3}$

$= -\frac{(\sqrt{3}-1)(1-\sqrt{3})}{2}$

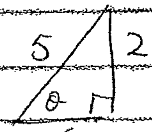
$= \frac{(1-\sqrt{3})^2}{2}$

$= \frac{(1-2\sqrt{3}+3)}{2}$

$= \frac{4-2\sqrt{3}}{2}$

$= \frac{\cancel{2}(2-\sqrt{3})}{\cancel{2}}$ ✓ *

$=$

3. $\sin\theta = \frac{2}{5}$ 

If θ is acute

$x^2 + 2^2 = 5^2$

$x^2 = 25 - 4$

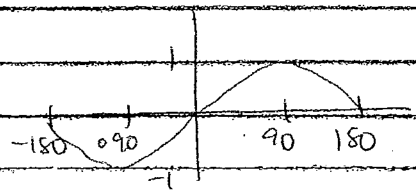
$= 21$

$x = \sqrt{21}$ ✓

$\cot\theta = \frac{1}{\tan\theta}$

$\tan\theta = \frac{2}{\sqrt{21}}$ ✓

$\cot\theta = +\frac{\sqrt{21}}{2}$ ✓



$\theta = 135^0, 225, 495, 585$

$x = 67\frac{1}{2}^0, 112\frac{1}{2}^0, 247\frac{1}{2}^0, 292\frac{1}{2}^0$ ✓

5. $\dfrac{\sin^2\theta}{1-\sin^2\theta} = \dfrac{\sin^2\theta}{\cos^2\theta}$ ✓

$= \tan^2\theta$ ✓

6. Prove $\tan A \sin A + \cos A = \sec A$

LHS $= \tan A \sin A + \cos A$

$= \dfrac{\sin^2 A}{\cos A} + \cos A$ ✓

$= \dfrac{\sin^2 A + \cos^2 A}{\cos A}$

$= \dfrac{1}{\cos A}$ ✓

$= \sec A$

$=$ RHS

7 a) $\sin x = 0 \quad -180^0 \le x \le 180^0$

$x = -180^0, 0^0, 180^0$ ✓✓

b) $\sin x = \frac{1}{2} \quad 0^0 \le x \le 360^0$

$x = 30^0 + x = 150^0$

c) $\cos^2 2x = \frac{1}{2} \quad 0 \le x \le 360^0$

let $\theta = 2x$

$0^0 \le \underset{\theta}{2x} \le 720$ ✓

$\cos^2\theta = \frac{1}{2}$

$\cos\theta = \pm \frac{1}{\sqrt{2}}$ ✓

$\cos\theta = \frac{1}{\sqrt{2}}$

$\theta = 45, 315, 405, 675$

$x = 22.5, 157\frac{1}{2}^0, 202\frac{1}{2}^0, 337\frac{1}{2}^0$

$\cos\theta = -\frac{1}{\sqrt{2}}$ ✓

d) $3\sec^2\theta + 2\tan\theta - 4 = 0$

$3(\tan^2\theta + 1) + 2\tan\theta - 4 = 0$

$3\tan^2\theta + 3 + 2\tan\theta - 4 = 0$

$3\tan^2\theta + 2\tan\theta - 1 = 0$

let $\tan\theta = x$

$3x^2 + 2x - 1 = 0$ ✓

$a = 3 \quad b = 2 \quad c = -1$

$\Delta = b^2 - 4ac$

$= 4 - 4 \times 3 \times -1$

$= 4 + 12$

$= 16$

$x = \dfrac{-2 \pm 4}{6}$

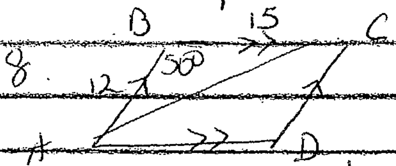
$x = -1$ or $x = \frac{2}{6}$ ✓

$= \frac{1}{3}$

$\tan\theta = -1 \qquad \tan\theta = \frac{1}{3}$

$\theta = 135^0, 315^0 \qquad \theta = 162^0, 342^0$

8.



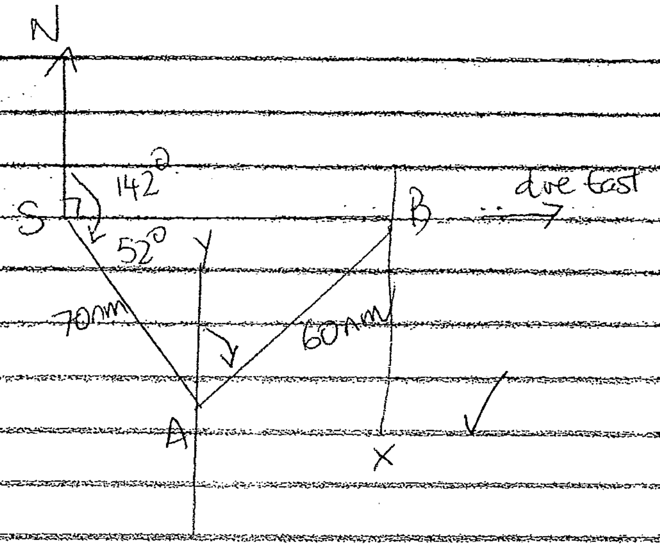
Area $ABC = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 12 \times 15 \sin 50^0$

$= 90 \sin 50^0$ ✓

Area $ABCD \doteq 138$ sq m ✓

9.



$\angle BSA = 142 - 90$

$= 52^\circ$ ✓

$\dfrac{70}{\sin B} = \dfrac{60}{\sin 50^\circ}$ ✓

$\dfrac{\sin B}{70} = \dfrac{\sin 50^\circ}{60}$

$\sin B = \dfrac{70 \sin 50^\circ}{60^\circ}$

$B \doteqdot 67^\circ$ ✓

$\angle ABX = 90 - \angle B$ ✓

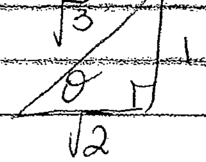
$= 23^\circ$

$= \angle YAB$ (alternate angles $YA \parallel BX$)

Bearing of the second course 023° or $N23^\circ E$.

10 θ is acute

$\sin\theta = \dfrac{1}{\sqrt{3}}$



$x^2 + 1^2 = (\sqrt{3})^2$

$x^2 = 3 - 1$

$= 2$

$x = \sqrt{2}$ ✓

$\tan\theta = \dfrac{1}{\sqrt{2}}$

$\sec\theta = \dfrac{1}{\cos\theta}$

$= \dfrac{\sqrt{3}}{\sqrt{2}}$ ✓

$\dfrac{\tan\theta}{1-\sec\theta} = \dfrac{\frac{1}{\sqrt{2}}}{1-\frac{\sqrt{3}}{\sqrt{2}}}$

$= \dfrac{\frac{1}{\sqrt{2}}}{\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}}}$

$= \dfrac{1}{\sqrt{2}} \div \dfrac{\sqrt{2}-\sqrt{3}}{\sqrt{2}}$

$= \dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}-\sqrt{3}}$

$= \dfrac{1}{\sqrt{2}-\sqrt{3}} \times \dfrac{\sqrt{2}+\sqrt{3}}{\sqrt{2}+\sqrt{3}}$

$= \dfrac{\sqrt{2}+\sqrt{3}}{2-3}$ ✓

$= -(\sqrt{2}+\sqrt{3})$

if θ is obtuse

$\tan\theta = -\frac{1}{2}$

$\sec\theta = -\dfrac{\sqrt{3}}{2}$

$\dfrac{\tan\theta}{1-\sec\theta} = \sqrt{2} - \sqrt{3}$ ✓

$\cancel{45}$