The trigonometric functions

TOPIC TEST

Time allowed: 1 hour **Total marks = 100**

SECTION I Multiple-choice questions

10 marks

Instructions
- This section consists of 10 multiple-choice questions
- Each question is worth 1 mark
- Fill in only ONE CIRCLE
- Calculators may be used

1 $\frac{3\pi}{5}$ radians = ?

Ⓐ 72° Ⓑ 108° Ⓒ 216° Ⓓ 300°

2 In radians, in terms of π, 450° = ?

Ⓐ $\frac{9\pi}{4}$ Ⓑ 3π Ⓒ $\frac{7\pi}{5}$ Ⓓ $\frac{5\pi}{2}$

3 Which is NOT a solution of the equation $2 \sin x = -1$?

Ⓐ $x = -\frac{\pi}{6}$ Ⓑ $x = \frac{5\pi}{6}$ Ⓒ $x = \frac{7\pi}{6}$ Ⓓ $x = \frac{11\pi}{6}$

4 The length of an arc of a circle of radius 8 cm subtended by an angle of $\frac{3\pi}{4}$ is:

Ⓐ $\frac{3\pi}{2}$ cm Ⓑ 3π cm Ⓒ 6π cm Ⓓ 24π cm

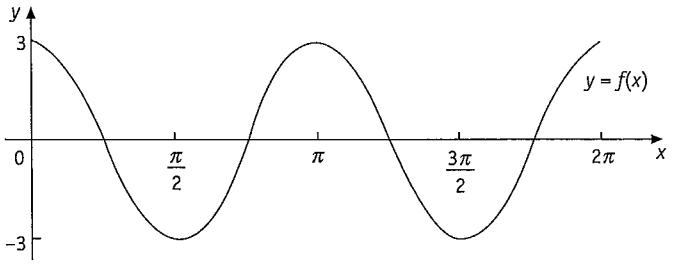
5 The diagram shows a sketch of the graph of $y = f(x)$, for $0 \leq x \leq 2\pi$. $f(x)$ = ?

Ⓐ $y = 3 \cos 2x$

Ⓑ $y = 2 \cos 3x$

Ⓒ $y = 3 \cos \frac{x}{2}$

Ⓓ $y = 2 \cos \frac{x}{3}$



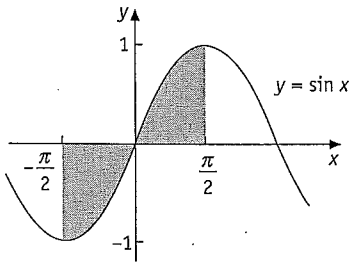
6 $\int 2\sec^2 \frac{x}{3}\, dx$ = ?

Ⓐ $\frac{2}{3} \tan \frac{x}{3} + C$ Ⓑ $2 \tan \frac{x}{3} + C$ Ⓒ $6 \tan \frac{x}{3} + C$ Ⓓ $\frac{2}{3} \tan x + C$

7 Which is correct? The shaded area in the diagram is equal to:

I $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x \, dx$

II $2\int_{0}^{\frac{\pi}{2}} \sin x \, dx$



Ⓐ I only Ⓑ II only

Ⓒ both I and II Ⓓ neither I nor II

8 The exact value of $\cos \frac{5\pi}{6}$ is?

Ⓐ $-\frac{\sqrt{3}}{2}$ Ⓑ $-\frac{1}{\sqrt{2}}$ Ⓒ $-\frac{1}{\sqrt{3}}$ Ⓓ $-\sqrt{3}$

9 The period of the graph of $y = 2 \sin \frac{x}{2}$ is?

Ⓐ π Ⓑ 2π Ⓒ $\frac{\pi}{2}$ Ⓓ 4π

10 $y = 4 \cos \frac{x}{3}$; $\frac{dy}{dx} = ?$

Ⓐ $-12 \sin \frac{x}{3}$ Ⓑ $-\frac{4}{3} \sin \frac{x}{3}$ Ⓒ $\frac{4}{3} \sin \frac{x}{3}$ Ⓓ $12 \sin \frac{x}{3}$

SECTION II

90 marks

Show all necessary working

11 Express in radians in terms of π:

2 marks each

a $80°$

b $36°$

12 Express in degrees:

2 marks each

a $\frac{\pi}{12}$

b $\frac{11\pi}{6}$

13 Find the value of $\sin \frac{3\pi}{5}$ correct to four decimal places. **2 marks**

14 Write down the exact value of:

a $\sin \frac{2\pi}{3}$ **b** $\tan \frac{11\pi}{6}$ **3 marks each**

15 Find all values of θ, $0 \leq \theta \leq 2\pi$, for which:

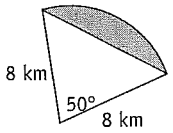
a $\cos \theta = \frac{\sqrt{3}}{2}$ **b** $\tan \theta = -1$ **4 marks each**

c $4 \sin^2 \theta = 3$ **4 marks**

16 Find the arc length of a sector of radius 9 cm with angle at the centre of 20°. **4 marks**

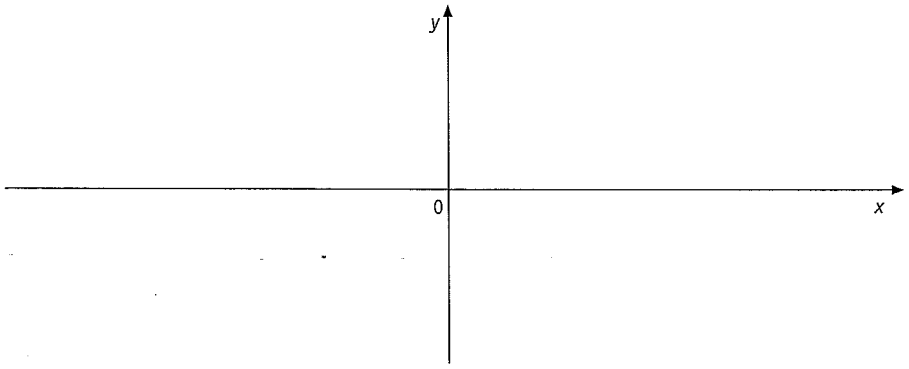
17 Find the area of a sector with radius 12 cm and angle at the centre of 240°. **4 marks**

18 Find the shaded area correct to one decimal place. **4 marks**

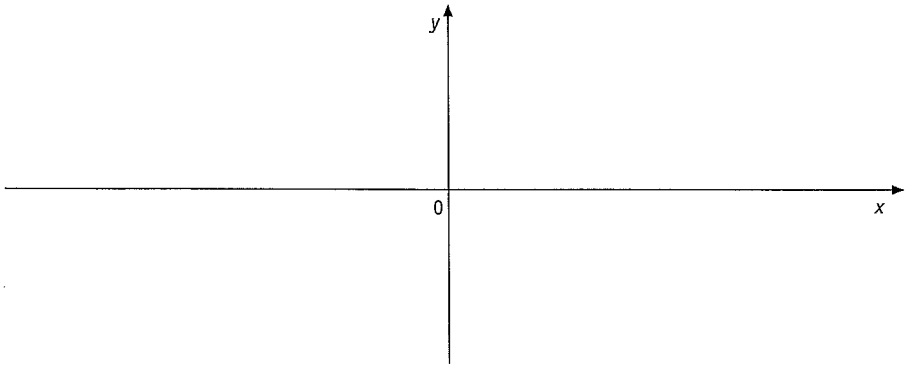


19 Sketch the graph of:

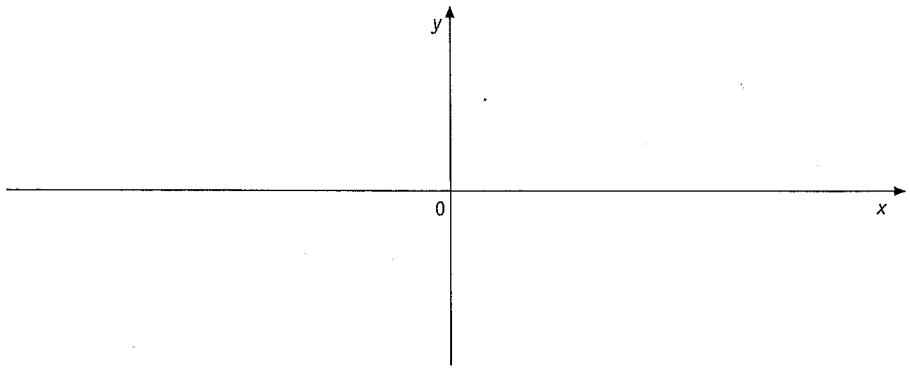
a $y = 2 \sin x$ **4 marks**



b $y = \cos 2x$ **4 marks**



c $y = 4 \tan x$ **4 marks**



20 Differentiate

a $2 \sin 4x$

b $-3 \cos \frac{x}{2}$ **3 marks each**

c $\tan\left(2x - \frac{\pi}{3}\right)$

d $x \sin x$ **3 marks each**

21 Find the equation of the tangent to the curve $y = \cos \pi x$ at the point when $x = \frac{1}{2}$ **4 marks**

22 Find:

a $\int \sin 2x \, dx$

b $\int 3 \cos \frac{x}{2} \, dx$ **3 marks each**

23 Find the exact value of each integral, expressing the answer as a single fraction:

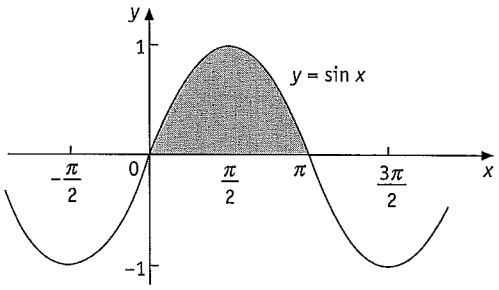
4 marks each

a $\int_0^{\frac{\pi}{4}} (1 - \cos x)\,dx$

b $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 x\,dx$

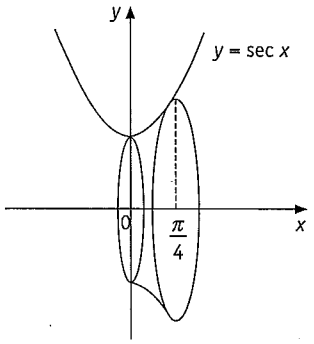
24 Find the area shaded in the diagram.

4 marks



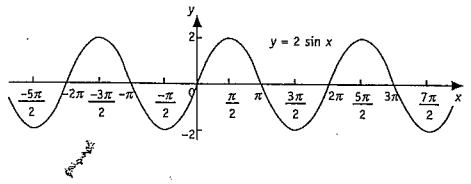
25 Find the volume of the solid formed when the section of the curve $y = \sec x$ between $x = 0$ and $x = \frac{\pi}{4}$ is rotated about the x-axis.

4 marks

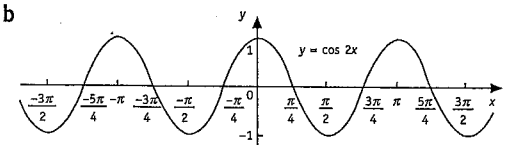


Pages 89-94 1 B 2 D 3 B 4 C 5 A 6 C 7 B 8 A 9 D 10 B 11 a $\frac{4\pi}{9}$ b $\frac{\pi}{5}$ 12 a 15° b 330° 13 0.9511 14 a $\frac{\sqrt{3}}{2}$ b $-\frac{1}{\sqrt{3}}$ 15 a $\frac{\pi}{6}$ or $\frac{11\pi}{6}$ b $\frac{3\pi}{4}$ or $\frac{7\pi}{4}$ c $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$ or $\frac{5\pi}{3}$ 16 π cm 17 96π cm^2 18 3.4 km^2

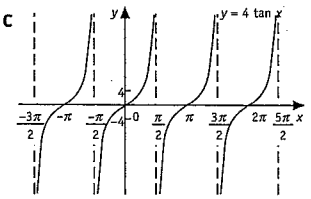
19 a



b



c



20 a $8\cos 4x$ b $\frac{3}{2}\sin\frac{x}{2}$ c $2\sec^2\left(2x-\frac{\pi}{3}\right)$ d $x\cos x+\sin x$ 21 $y=-\pi x+\frac{\pi}{2}$ 22 a $-\frac{1}{2}\cos 2x+C$ b $6\sin\frac{x}{2}+C$

23 a $\frac{\pi-2\sqrt{2}}{4}$ b $\frac{2\sqrt{3}}{3}$ 24 2 units^2 25 π units^3