

# The trigonometric functions

## Derivatives of $\sin x$ , $\cos x$ and $\tan x$ (1)

QUESTION 1 Complete:

As  $h \rightarrow 0$ ,

a  $\sin h \rightarrow$

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b  $\cos h \rightarrow$

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c  $\tan h \rightarrow$

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QUESTION 2 Differentiate:

a  $y = \sin x$

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b  $y = \cos x$

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c  $f(x) = \tan x$

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d  $y = \sin 3x$

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e  $y = 4 \cos x$

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f  $y = \tan \frac{x}{2}$

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g  $f(x) = 2 \sin 5x$

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h  $y = \frac{1}{2} \cos 2x$

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i  $y = \sin \pi x$

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j  $y = \sin (2x + 3)$

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k  $f(x) = 5 \tan \left( 3x - \frac{\pi}{4} \right)$

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l  $y = x - \cos x$

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m  $f(x) = \sin (x^2)$

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n  $y = x + \tan (x - 1)$

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o  $y = \cos (2 - 5x)$

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# The trigonometric functions

## Derivatives of $\sin x$ , $\cos x$ and $\tan x$ (2)



**QUESTION 1** Find the second derivative of:

**a**  $\sin 2x$

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**b**  $\cos \frac{x}{2}$

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**c**  $-4 \sin \pi x$

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**QUESTION 2** Find the value of the derivative at the given point:

**a**  $y = 3 \tan x$  at  $x = \frac{\pi}{4}$

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**b**  $y = \frac{1}{2} \sin 3x$  at  $x = \frac{\pi}{3}$

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**c**  $y = -\cos \pi x$  at  $x = 1$

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**QUESTION 3** Find the exact value of  $f'\left(-\frac{\pi}{3}\right)$  if:

**a**  $f(x) = 3 \sin x$

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**b**  $f(x) = \tan 4x$

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**c**  $f(x) = \frac{\cos x}{3}$

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# The trigonometric functions



## Derivatives of $\sin x$ , $\cos x$ and $\tan x$ (3)

QUESTION 1 Find the derivative of:

a  $y = x \cos x$

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b  $f(x) = x^2 \sin x$

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c  $f(x) = \tan^2 x$

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d  $y = \cos(x^2)$

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e  $y = \frac{\sin 2x}{x}$

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f  $y = \frac{1 - \cos x}{2x}$

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g  $y = \frac{1}{\cos x}$

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h  $f(x) = \operatorname{cosec} x$

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# The trigonometric functions

## Applications of differentiation



**QUESTION 1** Find the equation of the tangent to the curve  $y = 2 \sin 3x$  at  $x = \frac{\pi}{9}$

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**QUESTION 2** Find the equation of the normal to the curve  $y = \frac{\tan x}{2}$  at  $x = \frac{\pi}{4}$

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**QUESTION 3** Find all values of  $x$ ,  $0 \leq x \leq 2\pi$ , for which  $y = \cos x$  has gradient  $-\frac{1}{2}$

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**QUESTION 4** Show that the curve  $y = 2x - \sin x$  has no stationary points.

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**Page 81** 1 a 0 b 1 c 0 2 a  $\cos x$  b  $-\sin x$  c  $\sec^2 x$  d  $3 \cos 3x$  e  $-4 \sin x$  f  $\frac{1}{2} \sec^2 \frac{x}{2}$  g  $10 \cos 5x$  h  $-\sin 2x$  i  $\pi \cos \pi x$   
 j  $2 \cos (2x + 3)$  k  $15 \sec^2 \left( 3x - \frac{\pi}{4} \right)$  l  $1 + \sin x$  m  $2x \cos (x^2)$  n  $1 + \sec^2(x - 1)$  o  $5 \sin (2 - 5x)$

**Page 82** 1 a  $-4 \sin 2x$  b  $-\frac{1}{4} \cos \frac{x}{2}$  c  $4\pi^2 \sin \pi x$  2 a 6 b  $-1.5$  c 0 3 a 1.5 b 16 c  $\frac{\sqrt{3}}{6}$

**Page 83** 1 a  $-x \sin x + \cos x$  b  $x^2 \cos x + 2x \sin x$  c  $2 \tan x \sec^2 x$  d  $-2x \sin (x^2)$  e  $\frac{2x \cos x - \sin 2x}{x^2}$  f  $\frac{x \sin x + \cos x - 1}{2x^2}$   
 g  $\tan x \sec x \left( = \frac{\sin x}{\cos^2 x} \right)$  h  $-\cot x \operatorname{cosec} x \left( = -\frac{\cos x}{\sin^2 x} \right)$

**Page 84** 1  $3x - y + \sqrt{3} - \frac{\pi}{3} = 0$  2  $x + y - \frac{1}{2} - \frac{\pi}{4} = 0$  3  $x = \frac{\pi}{6}$  or  $\frac{5\pi}{6}$  4 ( $\cos x = 2$  has no solutions)