The trigonometric functions

Integrals (1)

QUESTION **1**   Find:

a $\int \sin x \, dx$

b $\int \cos x \, dx$

c $\int \sec^2 x \, dx$

d $\int 3 \cos x \, dx$

e $\int \sin 2x \, dx$

f $\int \frac{1}{2} \sec^2 4x \, dx$

g $\int -2 \sin \frac{x}{2} \, dx$

h $\int \sec^2 (x + 1) \, dx$

i $\int (1 - \cos x) \, dx$

j $\int \cos(3x - 2) \, dx$

k $\int \sin \pi x \, dx$

l $\int \frac{\sec^2 x}{2} \, dx$

QUESTION **2**   Find the exact value of:

a $\int_0^\pi \sin x \, dx$

b $\int_0^{\frac{\pi}{2}} \cos x \, dx$

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Integrals (2)

QUESTION **1**   Find the exact value of:

a $\displaystyle\int_0^{\frac{\pi}{4}} \sec^2 x\, dx$

b $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 2\sin x\, dx$

c $\displaystyle\int_0^1 \cos \pi x\, dx$

d $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{2}\sin 2x\, dx$

e $\displaystyle\int_{-\frac{\pi}{2}}^{0} \sec^2 \frac{x}{2}\, dx$

f $\displaystyle\int_0^{\pi} (x - \cos x)\, dx$

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Applications of integration (1)

QUESTION 1   Find the exact area bounded by the curve $y = 2 \sin x$ and the lines $x = \dfrac{\pi}{4}$, $x = \dfrac{3\pi}{4}$ and $y = 0$

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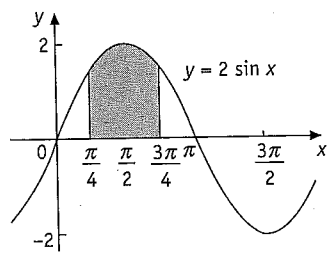
QUESTION 2

a   Complete the table of values for $f(x) = \sin^2 x$

x	0	$\dfrac{\pi}{6}$	$\dfrac{\pi}{3}$	$\dfrac{\pi}{2}$
$f(x)$				

b   Use the trapezoidal rule and these four function values to approximate $\displaystyle\int_0^{\frac{\pi}{2}} \sin^2 x \, dx$. Give the answer in terms of π.

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QUESTION 3

a   Differentiate $x \sin x$

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b   Hence find the exact value of $\displaystyle\int_0^{\frac{\pi}{2}} (x \cos x + \sin x) \, dx$

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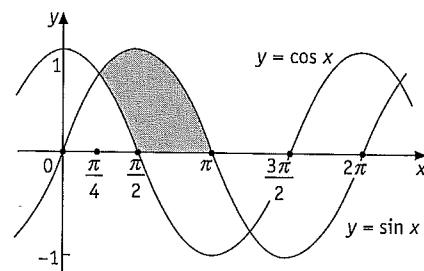
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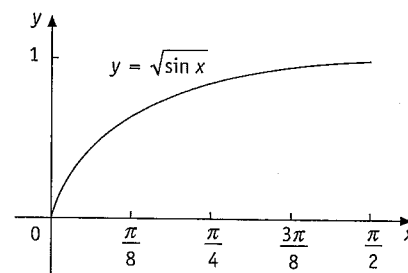
Applications of integration (2)

QUESTION 1   Find the area bounded by the curve $y = 2 \cos \dfrac{x}{2}$, the x-axis and the lines $x = 0$ and $x = \pi$
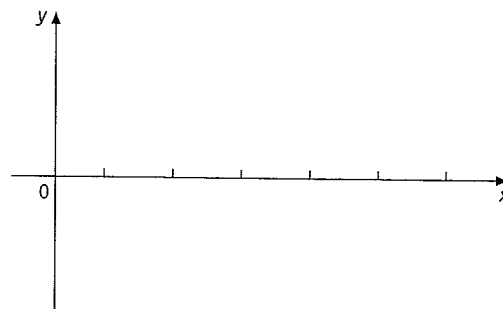
QUESTION 2   The diagram shows the graphs of $y = \sin x$ and $y = \cos x$. Find the shaded area.

QUESTION 3   The diagram shows the curve $y = \sqrt{\sin x}$, $0 \le x \le \dfrac{\pi}{2}$. Find the volume of the solid of revolution formed when this curve is rotated about the x-axis.

Page 85 **1** **a** $-\cos x + C$ **b** $\sin x + C$ **c** $\tan x + C$ **d** $3\sin x + C$ **e** $-\frac{1}{2}\cos 2x + C$ **f** $\frac{1}{8}\tan 4x + C$ **g** $4\cos\frac{x}{2} + C$

h $\tan(x+1) + C$ **i** $x - \sin x + C$ **j** $\frac{1}{3}\sin(3x-2) + C$ **k** $-\frac{1}{\pi}\cos\pi x + C$ **l** $\frac{\tan x}{2} + C$ **2** **a** 2 **b** 1

Page 86 **1** **a** 1 **b** $-1 + \sqrt{3}$ **c** 0 **d** 0 **e** 2 **f** $\frac{\pi^2}{2}$

Page 87 **1** $2\sqrt{2}$ units^2 **2** **a** 0, 0.25, 0.75, 1 **b** $\frac{\pi}{4}$ **3** **a** $x\cos x + \sin x$ **b** $\frac{\pi}{2}$

Page 88 **1** 4 units^2 **2** $\sqrt{2}$ units^2 **3** π units^3