Questions	**Marks**
(1) (a) Find the quadratic equation with roots 3 and -5.	1
(b) Solve the equation $3x^2 - 7x + 1 = 0$ leaving answers in surd form.	2
(c) (i) Find a, b, c if $3x^2 - 12x + 7 = a(x-b)^2 + c$	2
(ii) Hence, find the minimum value of $y = 3x^2 - 12x + 7$.	1
(d) Given that $3x^2 - 3kx + (2k-1) = 0$, find the value(s) of k if	
(i) the roots are equal.	3
(ii) one of the roots is 3	2
(iii) roots are reciprocal of one another.	2
(e) For the quadratic equation $5x^2 - 3x - 4 = 0$, find the value of:	4
(i) $\alpha + \beta$ (ii) $\alpha\beta$ (iii) $\frac{2}{\alpha} + \frac{2}{\beta}$	
(f) Find the value of k if $-x^2 + 6x + 3k < 0$	2
(g) (i) Solve $(x^2 - 3x)^2 - 2(x^2 - 3x) - 8 = 0$	4
(ii) Solve $4^x - 9.2^x + 8 = 0$	3
(2) (a) For the parabola $10y = x^2 - 5$, find the	
(i) coordinates of the vertex	1
(ii) focal length.	1
(iii) coordinates of the focus	1
(iv) equation of the directrix.	1

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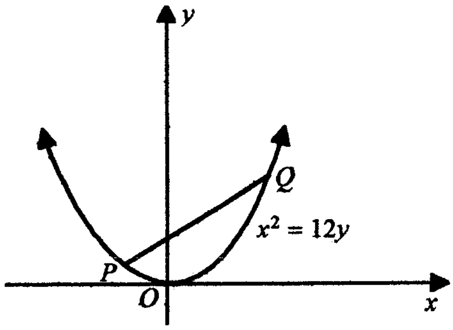
(b)



If $P\left(-2, \frac{1}{3}\right)$ and $Q(18, 27)$ are two points on the parabola $x^2 = 12y$,

(i) find the equation of the chord PQ **2**

(ii) show that the chord passes through the focus $(0, 3)$ **1**

(iii) find the equation of the tangent at P **2**

(iv) find the equation of the normal at P **2**

(v) if the tangent and normal intersect the y–axis at R and S respectively,

find the area of ΔPRS. **3**

2 UNIT TEST NUMBER 3

1996

SUGGESTED SOLUTIONS

QUESTION 1

(a) If the roots are 3 and −5 an equation could be

$(x-3)(x+5)=0$ or $x^2+2x-15=0$.

Either equation could also be multiplied by a constant. **1**

Alternative solution :

Using the formula

$x^2-(\text{sum of roots})x+(\text{product of roots})=0$,

$x^2-[3+(-5)]x+[3\times(-5)]=0$

Note : To avoid errors, put parentheses around negative numbers when they replace pronumerals.

$x^2-(-2)x+(-15)=0$

$x^2+2x-15=0$.

(b) Using the quadratic formula:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-7)\pm\sqrt{(-7)^2-4\times 3\times 1}}{2\times 3}$$ **1**

$$x=\frac{7\pm\sqrt{49-12}}{6}$$

$$x=\frac{7\pm\sqrt{37}}{6}.$$ **1** **Total = 2**

(c) (i) $3x^2-12x+7 = 3(x^2-4x)+7$ **1**

$= 3(x^2-4x+4)+7-12$

$= 3(x-2)^2-5$ **1**

Note : To complete the square in $(x^2-4x+\ldots)$, we add the square of half the coefficient of x, i.e. $\left(\frac{-4}{2}\right)^2=4$.
Also, since we added $3\times 4=12$, we must subtract 12 to maintain the equality of the expression.

Total = 2

Alternative solution :

$$3x^2 - 12x + 7 = a(x-b)^2 + c$$

$$= ax^2 - 2abx + ab^2 + c$$

Equate coefficients:

$a = 3$

$-2ab = -12 \quad \therefore b = 2$

$ab^2 + c = 7 \quad \therefore c = -5$

$\therefore 3x^2 - 12x + 7 = 3(x-2)^2 - 5$

(ii) As the minimum value of $(x-2)^2$ is 0

then the minimum value of

$3(x-2)^2 - 5$ is -5 . **1**

(d) Given $3x^2 - 3kx + (2k-1) = 0$

(i) Equal roots when $\Delta = 0$

$b^2 - 4ac = 0$ **1**

$(-3k)^2 - 4 \times 3 \times (2k-1) = 0$

$9k^2 - 24k + 12 = 0$

$3k^2 - 8k + 4 = 0$ **1**

$(3k-2)(k-2) = 0$

$k = \frac{2}{3}$ or $k = 2$ **1** **Total = 3**

(ii) Substitute $x = 3$ into the equation

$3(3)^2 - 3k(3) + (2k-1) = 0$ **1**

$27 - 9k + 2k - 1 = 0$

$26 = 7k$

$k = \frac{26}{7}$ **1** **Total = 2**

(iii) If roots are reciprocals then $\alpha\beta = 1$.

$\alpha\beta = \frac{c}{a}$

$\frac{2k-1}{3} = 1$ 1

$2k - 1 = 3$

$2k = 4$

$k = 2$ 1 **Total = 2**

(e) (i) $\alpha + \beta = -\frac{b}{a}$

$= -\frac{(-3)}{5}$

$= \frac{3}{5}$ 1

(ii) $\alpha\beta = \frac{c}{a}$

$= \frac{(-4)}{5}$

$= -\frac{4}{5}$ 1

(iii) $\frac{2}{\alpha} + \frac{2}{\beta} = \frac{2\beta + 2\alpha}{\alpha\beta}$

$= \frac{2(\alpha + \beta)}{\alpha\beta}$ 1

$= 2 \times \frac{3}{5} \div -\left(\frac{4}{5}\right)$ *Note :* From (i) and (ii).

$= -2 \times \frac{3}{5} \times \frac{5}{4}$

$= -\frac{3}{2}$ 1 **Total = 2**

(f) For negative definite $\Delta < 0$ and $k < 0$. 1

$b^2 - 4ac < 0$

$36 + 12k < 0$

$12k < -36$

$k < -3$ 1 **Total = 2**

(g) (i) $(x^2-3x)^2-2(x^2-3x)-8=0$

Note : Both of these questions relate to the syllabus statement "Equations reducible to quadratics".

Let $u=x^2-3x$

$u^2-2u-8=0$

$(u-4)(u+2)=0$ **1**

$u-4=0$ or $u+2=0$

$x^2-3x-4=0$ or $x^2-3x+2=0$ **1**

$(x-4)(x+1)=0$ or $(x-2)(x-1)=0$ **1**

Solutions are $x=4,\ 2,\ 1,\ -1$ **1** **Total = 4**

(ii) $(2^x)^2-9(2^x)+8=0$

Let $v=2^x$

$v^2-9v+8=0$

$(v-8)(v-1)=0$ **1**

$v-8=0$ or $v-1=0$

$2^x-8=0$ or $2^x-1=0$

$2^x=8$ or $2^x=1$ **1**

$2^x=2^3$ or $2^x=2^0$

Note : These are exponential equations and we equate the powers once the bases are the same.

$x=3$ or $x=0$ **1** **Total = 3**

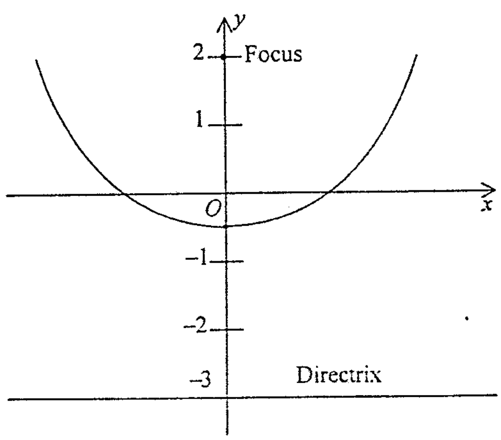
QUESTION 2

(a) Rewrite the parabola in the form $(x-h)^2=4a(y-k)$.

Note : $(x-h)^2=4a(y-k)$ has vertex at (h,k).

$x^2=10\left(y+\frac{1}{2}\right)$

i.e. $x^2=4\times\frac{5}{2}(y+\frac{1}{2})$



(i) Vertex is $(0, -\frac{1}{2})$. **1**

(ii) Focal length is $2\frac{1}{2}$ units. **1**

(iii) Coordinates of the focus are $(0, 2)$. **1**

(iv) Equation of the directrix is $y = -3$ **1** **Total = 4**

(b) (i) Use the two point formulae:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - \frac{1}{3} = \frac{27 - \frac{1}{3}}{18 + 2}(x + 2) \quad \textbf{1}$$

$$y - \frac{1}{3} = \frac{4}{3}(x + 2)$$

$$3y - 1 = 4x + 8$$

$$4x - 3y + 9 = 0 \quad \textbf{1}$$

Total = 2

Note : Students often learn the two point formula in the form $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$
OR: Find the gradient $m = \frac{y_2 - y_1}{x_2 - x_1}$ and use
$y - y_1 = m(x - x_1)$
where (x_1, y_1) can be either point.

(ii) The coordinates of the focus are $(0, 3)$, substitute this into the equation of the chord
$4x - 3y + 9 = 0$.

$$4(0) - 3(3) + 9 = 0 - 9 + 9$$

$$= 0.$$

Therefore the chord passes through the focus. **1**

Note : $(0, 3)$ satisfies the equation of the chord $4x - 3y + 9 = 0$.

(iii) Find the gradient of the tangent first.

$$y = \frac{x^2}{12}$$

$$\frac{dy}{dx} = \frac{2x}{12} = \frac{x}{6}$$

$$m = -\frac{1}{3} \quad \text{at } x = -2 \quad \textbf{1}$$

Equation of the tangent :

$$y - \frac{1}{3} = -\frac{1}{3}(x + 2)$$

Note : Using $y - y_1 = m(x - x_1)$.

$$3y - 1 = -x - 2$$

$$x + 3y + 1 = 0 \quad \textbf{1}$$

Total = 2

(iv) Gradient of the normal is 3, from part (iii).

Note : The result for gradients of perpendicular lines $m_1 m_2 = -1$ (the normal is perpendicular to the tangent).

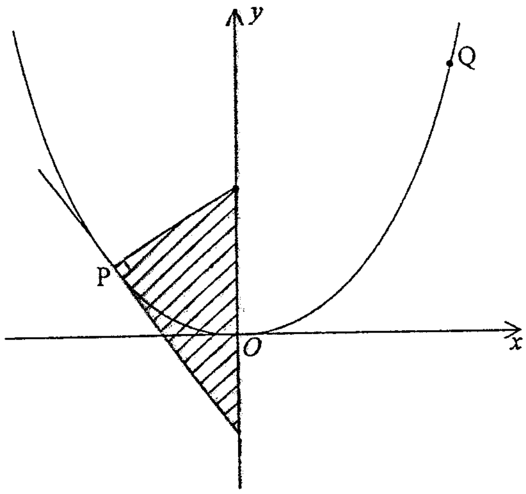
Equation of the normal: 1

$$y - \frac{1}{3} = 3(x+2)$$

$$y = 3x + 6\frac{1}{3}$$ 1 **Total = 2**

OR $9x - 3y + 19 = 0$

(v)



The y-intercept of the normal is $6\frac{1}{3}$

The y-intercept of the tangent is $-\frac{1}{3}$. 1

Area $= \frac{1}{2} \times \text{base} \times \text{height}$

Base is length on y-axis, and the height is the distance of P from the y-axis i.e. x-coordinate. 1

$$\text{Area} = \frac{1}{2} \times 6\frac{2}{3} \times 2$$

$$= 6\frac{2}{3} \text{ units}^2$$ 1 **Total = 3**