1. Simplify these surds:

   (a) $\sqrt{28}$,

   (b) $\sqrt{72}$,

   (c) $\sqrt{50} - \sqrt{18}$,

   (d) $3\sqrt{2} + 5\sqrt{8}$,

   (e) $3\sqrt{15} \times 5\sqrt{6}$,

   (f) $18\sqrt{150} \div 3\sqrt{3}$,

   (g) $3\sqrt{x} + \sqrt{9x}$,

   (h) $5a\sqrt{a} - \sqrt{a^3}$.

2. Expand and simplify:

   (a) $\left(5 + 2\sqrt{2}\right)\left(3 - 4\sqrt{10}\right)$,

   (b) $\left(2 + \sqrt{3}\right)^2$,

   (c) $\left(3 - 2\sqrt{5}\right)^2$,

   (d) $\left(4 - 7\sqrt{2}\right)\left(4 + 7\sqrt{2}\right)$

   (e) $\left(1 + \sqrt{m}\right)^2$,

   (f) $\left(3\sqrt{p} - 2\right)\left(3\sqrt{p} + 2\right)$.

3. Rationalize the denominator of:

   (a) $\dfrac{5}{\sqrt{5}}$,

   (b) $\dfrac{3\sqrt{2}}{5\sqrt{10}}$,

   (c) $\dfrac{1}{\sqrt{6} - \sqrt{5}}$,

   (d) $\dfrac{5 + \sqrt{2}}{3 - 2\sqrt{2}}$.

4. Simplify these expressions, leaving your answer as a single fraction without any surds in the denominator:

   (a) $2 + 5\sqrt{3} + \dfrac{1}{2 + 5\sqrt{3}}$,

   (b) $\dfrac{\sqrt{x} + \sqrt{2 + x}}{\sqrt{x} - \sqrt{2 + x}} + \dfrac{\sqrt{x} - \sqrt{2 + x}}{\sqrt{x} + \sqrt{2 + x}}$.

5. Given $x = 2\sqrt{3} - 1$, find $\dfrac{x}{x + \sqrt{2}}$ expressing your answer as a fraction with a rational denominator.

Question 1

(a) $\sqrt{28} = 2\sqrt{7}$

(b) $\sqrt{72} = 6\sqrt{2}$

(c) $\sqrt{50} - \sqrt{18} = 5\sqrt{2} - 3\sqrt{2}$
$= 2\sqrt{2}$

(d) $3\sqrt{2} + 5\sqrt{8} = 3\sqrt{2} + 10\sqrt{2}$
$= 13\sqrt{2}$

(e) $3\sqrt{15} \times 5\sqrt{6} = 15\sqrt{90}$
$= 15 \times 3\sqrt{10}$
$= 45\sqrt{10}$

(f) $18\sqrt{150} \div 3\sqrt{3} = 6\sqrt{50}$
$= 6 \times 5\sqrt{2}$
$= 30\sqrt{2}$

(g) $3\sqrt{x} + \sqrt{9x} = 3\sqrt{x} + 3\sqrt{x}$
$= 6\sqrt{x}$

(h) $5a\sqrt{a} - \sqrt{a^3} = 5a\sqrt{a} - a\sqrt{a}$
$= 4a\sqrt{a}$

Question 2

(a) $(5+2\sqrt{2})(3-4\sqrt{10}) = 15 - 20\sqrt{10} + 6\sqrt{2} - 8\sqrt{20}$
$= 15 - 20\sqrt{10} + 6\sqrt{2} - 16\sqrt{5}$

(b) $(2+\sqrt{3})^2 = 4 + 4\sqrt{3} + 3$
$= 7 + 4\sqrt{3}$

(c) $(3-2\sqrt{5})^2 = 9 - 12\sqrt{5} + 20$
$= 29 - 12\sqrt{5}$

(d) $(4-7\sqrt{2})(4+7\sqrt{2}) = 16 - 98$
$= -82$

(e) $(1+\sqrt{m})^2 = 1 + 2\sqrt{m} + m$

(f) $(3\sqrt{p}-2)(3\sqrt{p}+2) = 9p - 4$

Question 3

(a) $\dfrac{5}{\sqrt{5}} = \dfrac{5}{\sqrt{5}} \times \dfrac{\sqrt{5}}{\sqrt{5}}$
$= \dfrac{5\sqrt{5}}{5}$
$= \sqrt{5}$

(b) $\dfrac{3\sqrt{2}}{5\sqrt{10}} = \dfrac{3\sqrt{2}}{5\sqrt{10}} \times \dfrac{\sqrt{10}}{\sqrt{10}}$
$= \dfrac{3\sqrt{20}}{50}$
$= \dfrac{6\sqrt{5}}{50}$
$= \dfrac{3\sqrt{5}}{25}$

(c) $\dfrac{1}{\sqrt{6}-\sqrt{5}} = \dfrac{1}{\sqrt{6}-\sqrt{5}} \times \dfrac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}$
$= \dfrac{\sqrt{6}+\sqrt{5}}{1}$
$= \sqrt{6} + \sqrt{5}$

(d) $\dfrac{5+\sqrt{2}}{3-2\sqrt{2}} = \dfrac{5+\sqrt{2}}{3-2\sqrt{2}} \times \dfrac{3+2\sqrt{2}}{3+2\sqrt{2}}$
$= \dfrac{15+10\sqrt{2}+3\sqrt{2}+4}{9-8}$
$= 19 + 13\sqrt{2}$

Question 4

(a) $2+5\sqrt{3} + \dfrac{1}{2+5\sqrt{3}} = 2+5\sqrt{3} + \dfrac{1}{2+5\sqrt{3}} \times \dfrac{2-5\sqrt{3}}{2-5\sqrt{3}}$
$= 2+5\sqrt{3} + \dfrac{2-5\sqrt{3}}{4-75}$
$= \dfrac{2+5\sqrt{3}}{1} + \dfrac{2-5\sqrt{3}}{-71}$
$= \dfrac{71(2+5\sqrt{3})-(2-5\sqrt{3})}{71}$
$= \dfrac{142+355\sqrt{3}-2+5\sqrt{3}}{71}$
$= \dfrac{140+360\sqrt{3}}{71}$

(b) $\dfrac{\sqrt{x}+\sqrt{2+x}}{\sqrt{x}-\sqrt{2+x}} + \dfrac{\sqrt{x}-\sqrt{2+x}}{\sqrt{x}+\sqrt{2+x}}$
$= \dfrac{(\sqrt{x}+\sqrt{2+x})^2 + (\sqrt{x}-\sqrt{2+x})^2}{x-(2+x)}$
$= \dfrac{x+2\sqrt{x(2+x)}+2+x+x-2\sqrt{x(2+x)}+2+x}{-2}$
$= \dfrac{4x+4}{-2}$
$= -2x-2$

Question 5

$\dfrac{x}{x+\sqrt{2}} = \dfrac{2\sqrt{3}-1}{2\sqrt{3}-1+\sqrt{2}}$
$= \dfrac{2\sqrt{3}-1}{(2\sqrt{3}-1)+\sqrt{2}} \times \dfrac{(2\sqrt{3}-1)-\sqrt{2}}{(2\sqrt{3}-1)-\sqrt{2}}$
$= \dfrac{(2\sqrt{3}-1)^2 - \sqrt{2}(2\sqrt{3}-1)}{(2\sqrt{3}-1)^2 - 2}$
$= \dfrac{13-4\sqrt{3}-2\sqrt{6}+\sqrt{2}}{13-4\sqrt{3}-2}$
$= \dfrac{13-4\sqrt{3}-2\sqrt{6}+\sqrt{2}}{11-4\sqrt{3}} \times \dfrac{11+4\sqrt{3}}{11+4\sqrt{3}}$
$= \dfrac{143-44\sqrt{3}-22\sqrt{6}+11\sqrt{2}+52\sqrt{3}-48-8\sqrt{18}+4\sqrt{6}}{121-48}$
$= \dfrac{95+8\sqrt{3}-18\sqrt{6}+11\sqrt{2}-24\sqrt{2}}{73}$
$= \dfrac{95+8\sqrt{3}-18\sqrt{6}-13\sqrt{2}}{73}$