PROJECTILE MOTION

EXERCISE 6.9

1. A particle is projected at an angle of 45° and a velocity of 15 ms^{-1}. Neglecting air resistance and taking g as 10 ms^{-2}
   (a) derive the equation for the particle for (i) horizontal and (ii) vertical displacement (in exact form)
   (b) find the time taken to reach the ground

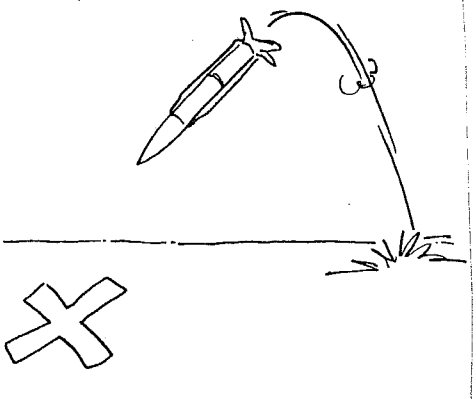
2. A gun is fired at an angle of 60° and with a velocity of 120 ms^{-1}. Assuming the acceleration due to gravity is 10 ms^{-2} and neglecting air resistance, find
   (a) the exact time taken for the bullet to reach its maximum height
   (b) the bullet's maximum height

3. A ball is thrown from a window that is 16 m from the ground. If the angle of projection is 60°, initial velocity is 5 ms^{-1}, and $g = 10\ ms^{-2}$, find
   (a) the time taken for the ball to land (to 1 decimal place)
   (b) how far the ball will land from the base of the building (to 1 decimal place)

4. Pham throws a ball in the air at a velocity of 8.7 ms^{-1} and at an angle of 55°. Neglecting air resistance and using $g = 9.8\ ms^{-2}$, find
   (a) the maximum height reached (to 2 decimal places)
   (b) how far away from Pham it will land (to 1 decimal place)

5. Michele throws a frisbee from the window of a building 15.3 m up. If the frisbee has an initial velocity of 8.8 ms^{-1} and is thrown at an angle of 73°, find the time taken for it to reach the ground (using $g = 10\ ms^{-2}$ and neglecting air resistance).

6. A missile is launched at an initial trajectory of 68° and a velocity of 1200 ms^{-1}. Neglecting air resistance and the curvature of the earth and taking the acceleration due to gravity as 9.8 ms^{-2}, calculate
   (a) the time taken for its flight (to the nearest minute)
   (b) how far away it will hit its target (to the nearest km)



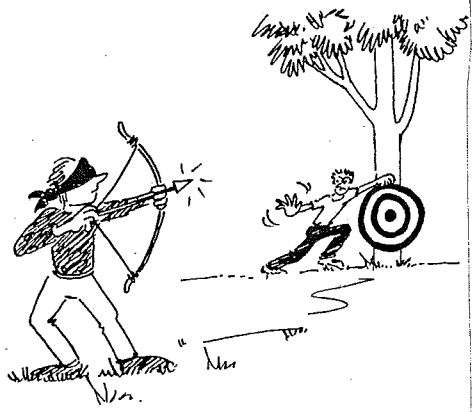
7. (a) A particle is projected upwards at an angle to the horizontal of α with velocity u. Derive the horizontal and vertical equations for acceleration, velocity and displacement for the flight of the particle, taking g as the acceleration due to gravity and neglecting air resistance.
   (b) If $u = 20\ ms^{-1}$, $\alpha = 60°$ and $g = 10\ ms^{-2}$, find the maximum height reached by the particle.

8. A particle is projected at a velocity of 16 ms^{-1} at an angle of elevation of θ.

Neglecting air resistance and using $g = 10\ \text{ms}^{-2}$, find the Cartesian equation of the displacement of the particle in terms of $\tan\theta$.

9. Find the equations of horizontal and vertical displacement of a particle with initial velocity $v\ \text{ms}^{-1}$ and angle of projection β if the particle is projected from a point h above ground level (use g for the acceleration due to gravity and neglect air resistance).

10. An arrow is fired at a velocity of $24\ \text{ms}^{-1}$ and is aimed at the centre of a target 1 m high and 35 m away. The air resistance is negligible for angles of projection less than 45°. At what angle should the arrow be fired? Use $g = 10\ \text{ms}^{-2}$.



11. A horizontal drainpipe 6 m above sea level empties stormwater into the sea. If the water comes out horizontally and reaches the sea 2 m out from the pipe, find the initial velocity of the water, correct to 1 decimal place. Let g be $10\ \text{ms}^{-2}$ and neglect air resistance.

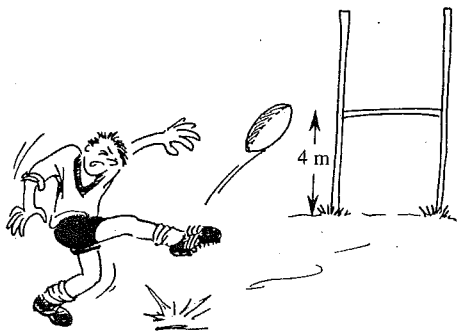
12. A particle is projected at an angle of 60° with velocity v. If it reaches the ground after 5.1 s, find the value of v correct to 2 decimal places (use $g = 9.8\ \text{ms}^{-2}$ and neglect air resistance).

13. A gun is aimed at a target on the ground 150 m away. If the initial velocity is $125\ \text{ms}^{-1}$, find the angles at which the gun could be fired to reach the target (use $g = 10\ \text{ms}^{-2}$).

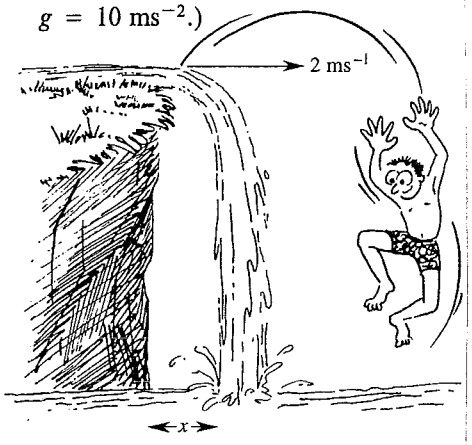
14. Jack stands at the window of a building 6.2 m above ground level. He throws his keys straight out of the window (horizontally) and hopes that his friend Tom, who is standing 10.4 m out from the base of the building, will catch them. Ignoring air resistance and using $10\ \text{ms}^{-2}$ for the acceleration due to gravity, find the velocity at which Jack needs to throw his keys, correct to 1 decimal place.

15. A rocket is fired straight up in the air at a fireworks display. When it reaches 28 m high, it explodes and is projected at an angle of 60° and a velocity of $30\ \text{ms}^{-1}$.
    (a) How long will it take the rocket to fall back to the ground, neglecting air resistance and taking $g = 10\ \text{ms}^{-2}$? Give your answer to the nearest second.
    (b) How far will the rocket land from its launching site (to 1 decimal place)?

16. A football is kicked at $12\ \text{ms}^{-1}$ and it just clears the goalpost 4 m high and 9 m away. Find the angle of projection through which the football is kicked (use $g = 10\ \text{ms}^{-2}$ and neglect air resistance).



17. A waterfall flows at 2 ms^{-1} over a vertical 5 m cliff. How far out from the cliff does it fall? (Use $g = 10\ ms^{-2}$.)



18. An object is projected with horizontal velocity 8 ms^{-1} and vertical velocity 5 ms^{-1}. Find the range of its flight (use $g = 10\ ms^{-2}$).

19. A stone is projected in the air at an angle of $\frac{\pi}{6}$ and a velocity of 15 ms^{-1}. How far from a window 20 m away does it land? (Use $g = 9.8\ ms^{-2}$.)

20. A particle is projected at an initial velocity of 10 ms^{-1}. If the horizontal component of the velocity is 6 ms^{-1}, find
    (a) the vertical component of velocity
    (b) the angle of projection
    (c) the maximum height of the particle (use $g = 10\ ms^{-2}$).

ANSWERS

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EXERCISE 6.9

1. (a) (i) $x = \frac{15\sqrt{2}t}{2}$ (ii) $y = -5t^2 + \frac{15\sqrt{2}t}{2}$ (b) $\frac{3\sqrt{2}}{2}$ s 2. (a) $6\sqrt{3}$ s (b) 540 m 3. (a) 2.3 s (b) 5.8 m 4. (a) 2.59 m (b) 7.3 m 5. 2.8 s
6. (a) 4 minutes (b) 102 km
7. (a) $\ddot{x} = 0, \dot{x} = u\cos a, x = ut\cos a$
$\ddot{y} = -g, \dot{y} = -gt + u\sin a,$
$y = -\frac{gt^2}{2} + ut\sin a$ (b) 15 m
8. $y = -\frac{5x^2}{256}(1 + \tan^2\theta) + x\tan\theta$
9. $x = vt\cos\beta, y = -\frac{gt^2}{2} + vt\sin\beta + h$
10. 20°34′ 11. 1.8 ms^{-1} 12. 28.86 ms^{-1}
13. 2°45′, 87°15′ 14. 9.3 ms^{-1} 15. (a) 6 s (b) 91.7 m 16. 63°6′ or 50°52′ 17. 2 m 18. 8 m
19. 0.12 m 20. (a) 8 ms^{-1} (b) 53°8′ (c) 3.2 m