Revision questions

41. If $g(x) = 4x - 9$, find x when $g(x) = 19$.

42. $f(x) = \begin{cases} 8x^2 - 2x + 3 & \text{when } x > 1 \\ x & \text{when } -1 \leqslant x \leqslant 1 \\ 9 & \text{when } x < -1 \end{cases}$.

Find $f(4) - f(-2) + f(-1)$.

43. Find the domain and range of $y = x^2 - 12$.

44. Sketch the region $x < 5$, $2x + 3y < 6$.

45. Find the domain and range of $x^2 + y^2 = 121$.

46. If $T(x) = 5 - x^4$ and $Q(x) = 8x - 9$, find the value of $T(-1) - Q(2)$.

47. Sketch the region defined by $y > \frac{1}{x}$ in the first quadrant.

48. If $h(t) = \begin{cases} 2 - t^3 & \text{if } t > 0 \\ t^2 + 1 & \text{if } t \leqslant 0 \end{cases}$, find the values of $h(3) + h(-4) - h(0)$.

49. Sketch $y = \sqrt{9 - x^2}$.

50. Sketch $y = \frac{1}{x^2}$.

Challenge questions

51. Find the domain of $f(x) = -\frac{1}{x^2 - 1}$.

52. Sketch the region $y \geqslant 2x - 5$, $y < x^2$.

53. If $f(x) = \begin{cases} 2x - 1 & \text{for } x \geqslant 0 \\ x + 3 & \text{for } x < 0 \end{cases}$, find $f(a^2)$.

54. Find the value(s) of x for which $f(x) = 0$ when $f(x) = x^2 - 3x - 1$ (give exact answers).

55. (a) Show that $\frac{x + 2}{x - 1} = 1 + \frac{3}{x - 1}$.

(b) Find the domain and range of $y = \frac{x + 2}{x - 1}$.

(c) Hence sketch the graph of $y = \frac{x + 2}{x - 1}$.

56. Sketch the region $y > \frac{1}{x + 5}$ in the first quadrant.

57. Sketch $y = |x| + 2x - 1$.

58. Find the domain of $y = \frac{1}{\sqrt{1 - x^2}}$.

59. Find $\lim_{x \to \infty} \frac{3\sqrt{x}}{2x + 1}$.

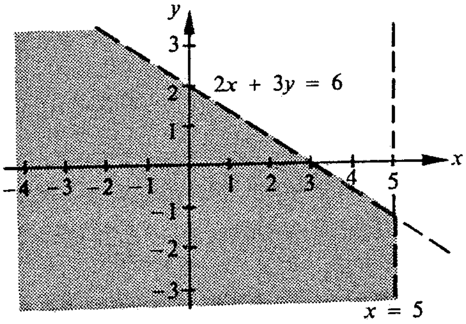
60. If $f(x) = \begin{cases} x^2 - 1 & \text{for } x \geqslant 1 \\ x + 1 & \text{for } -1 < x < 1 \\ 3 & \text{for } x \leq -1 \end{cases}$ sketch the function.

41. $x = 7$

42. $f(4) = 8(4)^2 - 2(4) + 3$
$= 123$ (since $4 > 1$)
$f(-2) = 9$ (since $-2 < -1$)
$f(-1) = -1$ (since $-1 \le -1 \le 1$)
So
$f(4) - f(-2) + f(-1) = 123 - 9 + (-1) = 113.$

43. *Domain*: all real x
Range: $y \ge -12$

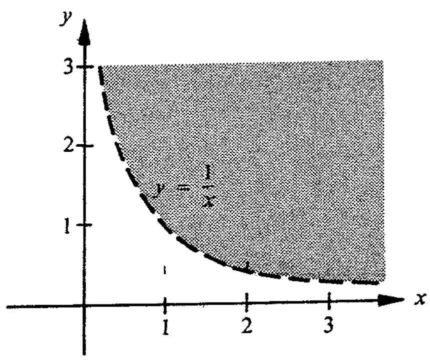
44. Sketch $x = 5$ (broken).
Region lies to the left of the line.
Sketch $2x + 3y = 6$ (broken).
Choose a point on one side of the line, say (0, 0):
$2x + 3y < 6$
$0 + 0 < 6$ (true).
So the region is on this side of the line.



45. *Domain*: $-11 \le x \le 11$
Range: $-11 \le y \le 11$

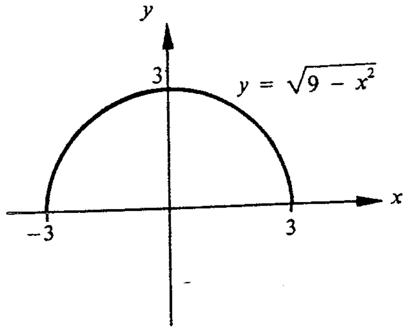
46. $T(-1) = 5 - (-1)^4 = 5 - 1 = 4$
$Q(2) = 8(2) - 9 = 7.$
So $T(-1) - Q(2) = 4 - 7 = -3.$

47.



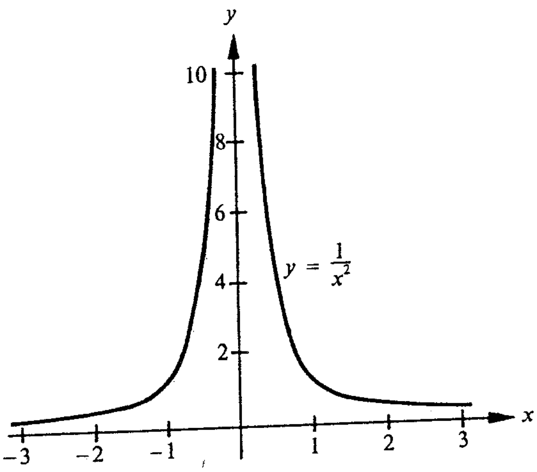
48. $h(3) = 2 - (3)^3 = -25$ (since $3 > 0$)
$h(-4) = (-4)^2 + 1 = 17$ (since $-4 \le 0$
$h(0) = 0^2 + 1 = 1$ (since $0 \le 0$).
So
$h(3) + h(-4) - h(0) = -25 + 17 - 1 = -9.$

49.



50.

x	-3	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{3}$	$\frac{1}{2}$	1	2	3
y	$\frac{1}{9}$	$\frac{1}{4}$	1	4	—	9	4	1	$\frac{1}{4}$	$\frac{1}{9}$



Challenge questions

51. *Domain*: all real numbers, $x \neq \pm 1$.

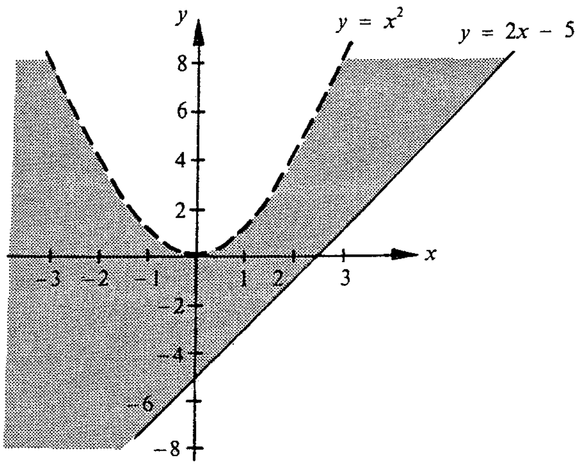
52. Sketch $y = 2x - 5$ (unbroken).
Choose a point on one side of the line, say (0, 0):

$$y \geq 2x - 5$$
$$0 \geq 0 - 5 \quad \text{(true)}.$$

So the region is on this side of the line.
Sketch $y = x^2$ (broken).
Choose a point on the outside of the curve, say $(0, -3)$:

$$y < x^2$$
$$-3 < 0 \quad \text{(true)}.$$

So the region is on the outside of the curve.

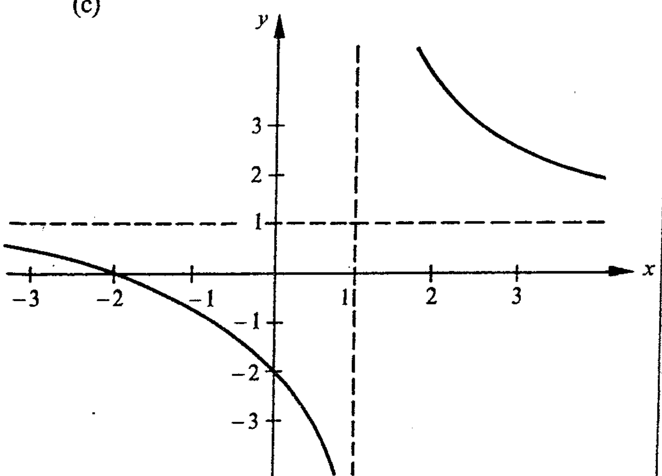


53. $f(a^2) = 2a^2 - 1$ (since $a^2 \geq 0$)

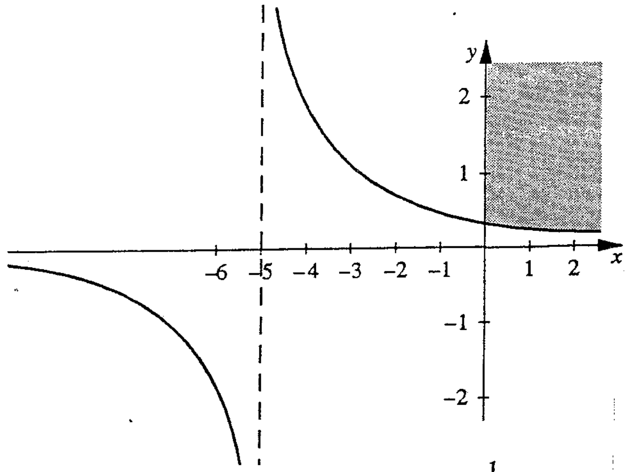
54. $x^2 - 3x - 1 = 0$
$a = 1, b = -3, c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-1)}}{2(1)}$$
$$= \frac{3 \pm \sqrt{9 + 4}}{2}$$
$$= \frac{3 \pm \sqrt{13}}{2}$$

55. (b) *Domain*: all $x, x \neq 1$
Range: all $y, y \neq 1$
(c)



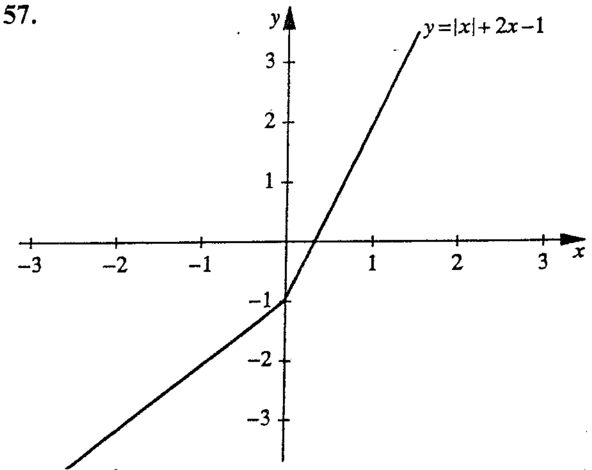
56. $y = \dfrac{1}{x + 5}$ has domain: all real $x \neq -5$, and range: all real $y \neq 0$.



Choose, say (2, 1): substitute in $y > \dfrac{1}{x + 5}$

$$1 > \frac{1}{2 + 5} \text{ (true)}$$

57.

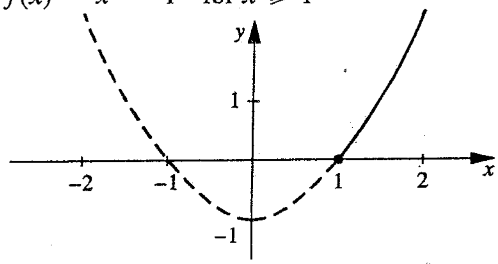


58. $1 - x^2 > 0$

$$1 > x^2$$
$$\therefore -1 < x < 1$$

59. 0

60. $f(x) = x^2 - 1$ for $x \geqslant 1$



$f(x) = x + 1$ for $-1 < x < 1$