EXERCISE 12.8

1. A combination lock has 4 dials, each with 10 digits. How many combinations are possible?
2. A certain type of serial number on a television set is made up of 5 numbers, followed by a letter. How many serial numbers of this type are available?
3. A personalised car numberplate can fit up to 6 letters on it. If I could use any letter of the alphabet, or a space, in any position, how many different combinations could be formed?
4. A Personal Identification Number (PIN) is made up of 4 digits. How many different PINs are possible?
5. Jan saw a car leaving the bank after a robbery. She remembered all of its numberplate except the last number. How many cars could there be with this numberplate?
6. A poker machine has 5 reels, each with 12 symbols on it. How many different combinations are possible?
7. A local library has 59 books on drama, 102 books on ballet and 87 books on gymnastics. I ask my friend to borrow a particular book on each of these subjects, but he loses my list. If he chooses 1 of each type of book at random, find the probability that he will choose the 3 books that I wanted.
8. A company that manufactures radios labels each radio with a serial number made up of 2 numbers and 3 letters. How many radios can have this type of serial number?
9. To win a trifecta in a race, a person has to pick the horses that come first, second and third in the race. For a certain race, Marie wishes to bet on every combination possible to win the trifecta. If there are 12 horses in the race, how many combinations will she bet on?
10. A security door unlocks when a person presses a certain combination of 6 numbers. If I try a particular combination at random, find the probability that I will be able to unlock the door.
11. The NSW and ACT postcodes all have 4 digits and start with 2. How many different postcodes are possible?
12. Many telephone numbers in Sydney have 7 digits. If the first digit is not allowed to be zero, how many telephone numbers are possible?
13. The game of Yahtzee involves tossing 5 dice.
    (a) How many ways can the 5 dice land?
    (b) What is the probability of tossing 5 sixes?
14. Wendy orders a 3 course dinner at a restaurant that offers 8 entrées, 5 main courses and 7 desserts. What is the probability that she has ordered the same combination as her friend?
15. A certain brand of car has a choice of 3 engine sizes and 7 colours, a choice of manual or automatic, and a choice of a sunroof or air conditioning. How many different combinations are possible?

EXERCISE 12.8

1. 10 000 2. 2 600 000 3. 387 420 489 4. 10 000 5. 10 6. 248 832 7. $\frac{1}{523\,566}$ 8. 1 757 600 9. 1320 10. $\frac{1}{1\,000\,000}$ 11. 1000 12. 9 000 000 13. (a) 7776 (b) $\frac{1}{7776}$ 14. $\frac{1}{280}$ 15. 84

EXERCISE 12.9

1. Evaluate, then check answers on the calculator
   (a) ${}^6P_4$
   (b) ${}^7P_6$
   (c) ${}^9P_1$
   (d) ${}^5P_3$
   (e) ${}^8P_6$

2. If I have 10 cards, each labelled with a different number from 1 to 10, find how many numbers are possible if selecting (without replacement)
   (a) 2 cards
   (b) 6 cards
   (c) 5 cards
   (d) 3 cards
   (e) 8 cards

3. A 3 digit number is to be made from the digits 2, 3, 4, 5, 6 and 7.
   (a) How many numbers can be made if no digit may be used more than once in the same number?
   (b) In how many ways can an even number be made from these digits?
   (c) How many numbers over 600 can be made?

4. A 4 digit number is to be made from the digits 1 to 9, with no digit allowed more than once in the same number.
   (a) How many numbers can be made?
   (b) In how many ways can an odd number be made?
   (c) How many numbers less than 3000 can be made?

5. (a) How many numbers can be made using the digits 1 to 4, with no digit allowed more than once? (The number can have any number of digits.)
   (b) How many numbers less than 100 can be made?

6. How many 3 letter combinations can be made from the word SWITZERLAND?

7. How many arrangements of the digits 1, 2, 3 and 4 can be made without using any digit more than once in the same number?

8. How many 3 digit numbers can be made from the digits 1, 2, 3, 4 and 5 without using any digit twice in the same number?

9. In how many ways can a group of 7 people be arranged
   (a) in a straight line
   (b) around a circle?

10. In how many ways can a group of 4 people be arranged
    (a) in a straight line
    (b) around a circle?

11. A queue has 4 boys and 4 girls standing in line. Find how many different arrangements of the line are possible if
    (a) the boys and girls can stand anywhere in the line
    (b) the boys and girls alternate
    (c) 2 particular girls wish to stand together
    (d) all the boys stand together.
    (e) Also find the probability that 3 particular people will be in the queue together if the queue forms randomly.

12. A table has 4 boys and 4 girls sitting around it.
    (a) Find the number of ways of sitting possible if the boys and girls can sit anywhere around the table.
    (b) If the seating is arranged at random, find the probability that (i) 2 particular girls will sit together, and (ii) all the boys will sit together.

13. How many ways can 7 people be arranged in a
    (a) line?
    (b) circle?

14. At a dinner party 6 people sit around a table.
    (a) How many different arrangements are possible?
    (b) Find the probability that if seating is at random, 2 friends will sit apart.
    (c) How many ways can the seating be arranged so that 3 particular people will sit together?

15. (a) Nine beads are arranged randomly in a line. How many arrangements are possible?
    (b) If the beads are placed in a circle, how many ways can this be done?
    (c) The beads are arranged on a bracelet. How many different ways are possible?
16. Find the probability that in a circle of 20 people, 2 particular people will be together.
17. How many different arrangements can be made from the following words? Note that the arrangements need not be proper words.
    (a) MATHS
    (b) WORD
    (c) ELEPHANT
    (d) POPULAR
    (e) SAUSAGE
    (f) TEPEE
    (g) BLACKBOARD
    (h) PERCENTAGE
    (i) ENGINEERING
    (j) SUPERMARKET
18. Find out how many arrangements are possible from the word STUDIO if
    (a) any grouping of letters is allowed
    (b) the T and D are together
    (c) the vowels are together
    (d) the vowels and consonants alternate
    (e) the letter S is not the first letter
    (f) the letter O is not first or last
19. In how many ways can 10 boys be arranged in a line if
    (a) the first boy in the line is always the same
    (b) the first boy and the last boy in the line are always the same?
20. Twelve differently coloured beads are arranged around a necklace. How many different arrangements are possible?

EXERCISE 12.9

1. (a) 360 (b) 5040 (c) 9 (d) 60 (e) 20 160
2. (a) 90 (b) 151 200 (c) 30 240 (d) 720 (e) 1 814 400
3. (a) 120 (b) 60 (c) 40
4. (a) 3024 (b) 1680 (c) 672
5. (a) 64 (b) 16
6. 990
7. 24
8. 60
9. (a) 5040 (b) 720 (c) 10 080 (d) 2880 (e) $\frac{3}{28}$
10. (a) 24 (b) 6
11. (a) 40 320 (b) 1152
12. (a) 5040 (b) (i) $\frac{2}{7}$ (ii) $\frac{4}{35}$
13. (a) 5040 (b) 720
14. (a) 120 (b) $\frac{3}{5}$ (c) 36
15. (a) 362 880 (b) 40 320 (c) 20 160
16. $\frac{2}{19}$
17. (a) 120 (b) 24 (c) 20 160 (d) 2520 (e) 1260 (f) 20 (g) 907 200 (h) 604 800 (i) 277 200 (j) 9 979 200
18. (a) 720 (b) 240 (c) 144 (d) 72 (e) 600 (f) 480
19. (a) 362 880 (b) 40 320
20. 19 958 400

EXERCISE 12.10

1. In a class of 20 students, 2 are selected as class prefects. If the selection is made at random, in how many ways is the selection possible?

2. A group of 8 tennis players all have an equal chance of being chosen in a team. If the team can only have 5 players, in how many ways can the team be selected?

3. In a group of 6 apprentices, a team of 4 is put on a special job. If the selection is made at random, in how many different ways can the selection of the team be made?

4. A debating team of 4 is chosen from a class of 15. How many different combinations are possible
   (a) if there are no restrictions on who is in the team
   (b) if 1 particular person is to be included in the team
   (c) if 2 particular people are to be included?

5. A committee of 3 women and 5 men is to be selected at random from 7 women and 7 men.
   (a) Find the number of ways that the committee can be chosen.
   (b) Among the group is a married couple. Find the probability that they will both be selected.

6. Out of 25 students who study drama, 3 are chosen to be in a play.
   (a) In how many ways can this be done if the selection is random?
   (b) A brother and sister both do drama. Find the probability that neither of them will be selected for the play.

7. There are 6 books about Ancient Rome on a certain shelf in the library. If there are 20 books on the shelf altogether and I choose 6 at random, find the probability that they will all be about Ancient Rome.

8. An excursion is arranged for a class of 31 students. However, there is only room for 20 students in the bus. If 20 students are selected at random from the class, in how many ways can this be done?

9. To win Lotto, you choose 6 numbers out of a total of 44.
   (a) In how many ways can this be done?
   (b) What is the probability of winning first prize in Lotto if you play 4 games?
   (c) Find the probability of winning Lotto if you play 100 games.

10. To play the Australian Soccer Pools you choose 6 numbers out of 36. What is the probability of winning if you play 10 games?

11. A committee of 3 people is formed from a group of 9 people.
    (a) In how many different ways can the committee be formed?
    (b) If the group consists of 6 men and 3 women, how many ways can the committee be formed with 2 men and 1 woman?

12. A team of 3 girls and 3 boys is chosen from a group of 15 girls and 12 boys to represent their school. In how many different ways can the team be formed?

13. There are 9 people who are applying for 4 jobs.
    (a) If the jobs are allocated randomly, how many different combinations of people are possible for the 4 jobs?
    (b) Out of the 9 people, 5 have their HSC. If 3 jobs require the HSC, how many ways can the 4 jobs be allocated?

14. Thirty people apply for a special housing package, but only 12 packages are available, and are allocated randomly.
    (a) What is the probability that a particular person applying will get a house?
    (b) Out of the 12 packages, 5 are in Sydney and 7 are in the Blue Mountains. If 19 people apply for the Sydney houses and the rest apply for the Blue Mountains, in how many ways can the houses be allocated?

15. Of a group of 16 children at preschool, only 5 are allowed on the climbing equipment at one time.
    (a) How many different combinations of children are possible?
    (b) Out of the 16 children, 7 are under 3 years old. If 2 children under 3 years and 3 over 3 years are allowed on the climbing equipment, how many ways can this be done?
    (c) Find the probability that Allan, who is 2 years old, and his friend Hannah, who is 4 years old, will both be included in the group chosen to play on the climbing equipment.

16. A school committee is to be made up of 5 teachers, 4 students and 3 parents.
    (a) If 12 teachers, 25 students and 7 parents apply to be on the committee, which is chosen at random, how many possible committees could be formed?
    (b) If Jan and her mother both apply, find the probability that both will be chosen for the committee.

EXERCISE 12.10

1. 190 2. 56 3. 15 4. (a) 1365 (b) 364 (c) 78
5. (a) 735 (b) $\frac{15}{49}$ 6. (a) 2300 (b) $\frac{77}{100}$ 7. $\frac{1}{38\,760}$
8. 84 672 315 9. (a) 7 059 052 (b) $\frac{1}{1\,764\,763}$
(c) $\frac{25}{1\,764\,763}$ 10. $\frac{5}{973\,896}$ 11. (a) 84 (b) 45
12. 100 100 13. (a) 126 (b) 40 14. (a) $\frac{1}{86\,493\,225}$
(b) 3 837 240 15. (a) 4368 (b) 1764 (c) $\frac{2}{21}$
16. (a) 350 658 000 (b) $\frac{3036}{44\,275}$ 17. (a) 10 (b) 6
18. (a) 560 (b) $\frac{1}{10}$ 19. (a) 350 (b) 140
(c) $\frac{12}{35}$ 20. (a) 5 852 925 (b) 66 (c) $\frac{2}{45}$

CHALLENGE EXERCISE PERMUTATIONS, COMBINATIONS AND PROBABILITY

1. In a group of 35 students, 25 go to the movies and 15 go to the football. If all the students like at least one of these activities, find the probability that a student chosen at random will
   (a) go to both the movies and the football
   (b) only go to the movies

2. In a train compartment, there are 8 seats, with 4 facing the front and 4 facing backwards.
   (a) If 5 people sit in the compartment, in how many ways can they be arranged?
   (b) If 2 of the people do not like sitting backwards, in how many ways can the 5 people be arranged?
   (c) Find the probability that 2 particular people will sit opposite each other if seating is arranged at random.

3. In how many different ways can the word MISSISSIPPI be arranged?

4. A certain soccer team has a probability of 0.5 of winning a match and a probability of 0.2 of drawing. If the team plays 2 matches, find the probability that it will
   (a) draw both matches
   (b) win at least 1 match
   (c) not win either match

5. A game of poker uses a deck of 52 cards with 4 suits (hearts, diamonds, spades and clubs). Each suit has 13 cards, consisting of an ace, cards numbered from 2 to 10, a jack, queen and king. If a person is dealt 5 cards find the probability of getting
   (a) four aces
   (b) a flush (all cards the same suit)

6. I throw a coin k times. Find an expression to describe the probability of throwing
   (a) at least 1 tail
   (b) $(k - 3)$ heads
   (c) 9 tails

7. The probability of an egg hatching out at a certain farm is 73%.
   (a) If there are 20 eggs, find the probability that they will all hatch.
   (b) Find the probability that 17 eggs will hatch.
   (c) Find the probability that k eggs will hatch.

8. Twelve students sit at a round table.
   (a) How many ways can they be arranged?
   (b) If 4 students wish to sit together, how many seating arrangements can be made?
   (c) Find the probability that 2 friends will be separated from each other if the seating arrangement is random.

9. A boat has 2 seats facing the bow and 2 seats facing aft. Four people are sitting in the boat.
   (a) How many ways can they be arranged in the seats?
   (b) One person does not like to sit facing aft. How many ways can the seating be arranged?

10. A squad of 8 is chosen at random from 3 baseball teams with 10 players in each team.
   (a) In how many ways can this squad be selected?
   (b) If 5 of the squad are chosen from the A team, and 2 from the B team, and 1 is chosen from the C team, how many ways can the squad be formed?
   (c) Find the probability that Joe from the B team and Dan from the A team will be chosen.

11. There are n seats around a circular table, and n people are arranged randomly around the table.
   (a) In how many ways can they be arranged?
   (b) What is the probability of 2 particular people sitting together?
   (c) Show that the probability of 3 particular people sitting together is $\frac{6}{(n-1)(n-2)}$.
   (d) What is the probability of k people sitting together?

12. A set of cards contains 52 cards with 4 suits (hearts, diamonds, spades and clubs). Each suit has 13 cards, consisting of an ace, jack, queen, king and cards numbered from 2 to 10.
   If a card is drawn out at random, find the probability that it will be
   (a) an ace or a heart
   (b) a diamond or an odd number
   (c) a jack or a spade

13. Bill does not select the numbers 1, 2, 3, 4, 5 and 6 for Lotto as he says this combination would never win. Is he correct?

CHALLENGE EXERCISE 12

1. (a) $\frac{1}{7}$ (b) $\frac{4}{7}$ 2. (a) 6720 (b) 1440 (c) $\frac{1}{7}$
3. 34 650 4. (a) 0.04 (b) 0.75 (c) 0.25
5. (a) $\frac{1}{54\,145}$ (b) $\frac{33}{10\,660}$ 6. (a) $1 - \frac{1}{2^k}$ (b) ${}^kC_{k-3}\frac{1}{2^k}$
(c) ${}^kC_9\frac{1}{2^k}$ 7. (a) 0.18% (b) 10.7%
(c) ${}^{20}C_k(0.73)^k(0.27)^{20-k}$ 8. (a) 39 916 800
(b) 967 680 (c) $\frac{9}{11}$ 9. (a) 24 (b) 12
10. (a) 5 852 925 (b) 113 400 (c) $\frac{1}{10}$

11. (a) $(n-1)!$ (b) $\frac{2}{n-1}$
(c) Probability $= \frac{3!(n-3)!}{(n-1)!}$
$= \frac{3 \times 2 \times 1 \times (n-3)(n-4)\ldots}{(n-1)(n-2)(n-3)(n-4)\ldots}$
$= \frac{6}{(n-1)(n-2)}$
(d) $\frac{k!(n-k)!}{(n-1)!} = \frac{k!}{(n-1)(n-2)(n-3)\ldots(n-k+1)}$

12. (a) $\frac{4}{13}$ (b) $\frac{25}{52}$ (c) $\frac{4}{13}$
13. No—any combination of numbers is equally likely to win.

Ex 12.9

2. a) $^{10}P_2 = 45$ ✓

b) $^{10}P_6 = 210$

c) $^{10}P_2 = 252$

d) $^{10}P_3 = 120$

e) $^{10}P_8 = 45$

3.

a) $6 \times 5 \times 4 = 120$ ✓

(b) $3(5 \times 4 \times 1) = 60$ ✓

(c) $2 \times 5 \times 4 = 40$ ✓

4. $9 \times 8 \times 7 \times 6 = 3024$ ✓

b) ① 2. ③ 4 ⑤ 6 ⑦ 8 9

$5(8 \times 7 \times 6 \times 1) = 1680$ ✓

c) $2 \times 8 \times 7 \times 6 = 672$ ✓

5a.) 1. 2 3 4.

1 digit = 4

2 digit = 4×3 ✓

3 digit = $4 \times 3 \times 2$

4 digit = 4!

Total = 64. ✓

6.) SWITZERLAND

$11 \times 10 \times 9 = 990$ ✓

7. 1 2 3 4.

4! = 24 ✓

8. 1 2 3 4 5

$3 \times 2 \times 3 = 60$ ✓

9.) 7! = 5040 ✓

$\frac{7!}{7} = 720$ ✓

11. 8! = 40 320 ✓

(b) $2 \times 4 \times 4 \times 3 \times 3 \times 2 \times 2 \times 1 \times 1 = 576 \times 2$ (Boys or Girl can come first)

$= 1152$

(c) GG GG BBBB $= 7! \times 2!$

$= 10080$ ✓

(d) BBBB GGGG $= 5! \times 4!$

$= 2880$ ✓

(e.) $3! \times 5! = 4320$

Probability $= \frac{4320}{10080}$ ✓

$= \frac{3}{28}$ ✓

12. a.) $\frac{8!}{8} = 5040$ ✓

b.) $\frac{7!}{7} \times 2! = 1440$ ✓

Probability $= \frac{2}{5}$ ✓

ii) $\frac{5!}{5} \times 4! = 576$ ✓

Probability $= \frac{4}{35}$ ✓

13. $7! = 5040$ ✓

b) $6! = 720$ ✓

14.) $\frac{6!}{6} = 5! = 120$ ✓

(b) Complement.

$\frac{5!}{5} \times 2! = 48$ ✓

Prob $= \frac{2}{5}$ ✓

∴ Prob $= 3/5$

c) $\frac{4!}{4} \times 3! = 36$ ✓

15.) $9! = 362\,880$ ✓

b) $\frac{9!}{9} = 40320$ ✓

× (c) $\frac{40\,320}{2} = 20160$ ✓

16. $\frac{19!}{19} \times 2! = 1.28 \times 10^{16}$

Probability $= 2/19$ ✓

17.)

a) BLACKBOARD $= \frac{10!}{2!2!}$

$= 907\,200$ ✓

h.) PERCENTAGE $= \frac{10!}{3!}$ ✓

$= 604\,800$

i) ENGINEERING

$= \frac{11!}{3!2!2!3!}$

$= 277\,200$ ✓

j.) SUPERMARKET $= \frac{11!}{2!2!}$

$= 9\,979\,200$ ✓

18. STUDIO

a) $6! = 720$ ✓

b.) (TD) SUIO $= 5! \times 2!$

$= 240$ ✓

c) (VIO) STD $= 4! \times 3!$

$= 144$ ✓

d.) $2(3 \times 3 \times 2 \times 2 \times 1 \times 1) = 36$

$= 72$ ✓

e.) $5 \times 5 \times 4 \times 3 \times 2 \times 1 = 800$ ✓

× (f.)

5	4	3	2	1	4

$= 480$ ✓

19. fill 1st & last first

$1 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$9! = 362\,880$ ✓

b) $1 \times 8 \times 7 \times 6 \times 5 \times 4^{3 \times 2 \times 1} \times 1$

$= 40\,320$ ✓

20. $\frac{12!}{12.2} = 19\,958\,400$ ✓

Ex 12.10.

1. ${}^{20}C_2 = 190$ ✓

2. ${}^{8}C_5 = 56$ ✓

3. ${}^{6}C_4 = 15$ ✓

4. ${}^{15}C_4 = 1365$ ✓

(b) $1 \times {}^{14}C_3 = 364$ ✓

(c) ${}^{13}C_2 = 78$ ✓

5. ${}^{7}C_3 \times {}^{7}C_5 = 735$ ✓

(b) $\frac{3}{7} \times \frac{5}{7} = \frac{15}{49}$ ✓

6. ${}^{25}C_3 = 2300$ ✓

(b) $\frac{22}{25} \times \frac{21}{24} = \frac{77}{100}$ ✓

7. $\frac{6}{20} \times \frac{5}{19} \times \frac{4}{18} \times \frac{3}{17} \times \frac{2}{16} \times \frac{1}{15}$ ✓

$= \frac{1}{38760}$ ✓

8. ${}^{31}C_{20} = 84\,672\,315$ ✓

9. ${}^{44}C_6 = 7\,059\,052$ ✓

(b) $\frac{4}{7059052} = \frac{1}{1764763}$ ✓

(c) $\frac{25}{1764763}$ ✓

10. ${}^{36}C_6 = 1\,947\,762$ ✓

Probability $= \frac{5}{973896}$ ✓

11. ${}^{9}C_3 = 84$ ✓

(b) ${}^{3}C_1 \times {}^{6}C_2 = 45$ ✓

12. ${}^{15}C_3 \times {}^{12}C_3 = 100100$ ✓

13. ${}^{9}C_4 = 126$ ✓

(b) ${}^{5}C_3 \times {}^{4}C_1 = 40$ ✓

${}^{5}C_4 = 5$

Total $= 45$

14.

(a) ${}^{30}C_{12} = 86\,493\,225$

Probability $= \frac{1 \times {}^{29}C_{11}}{{}^{30}C_{12}}$ ✓

$= \frac{2}{5}$ ✓

(b) ${}^{19}C_5 \times {}^{11}C_7 = 38\,372\,440$ ✓

15. ${}^{16}C_5 = 4368$ ✓

(b) ${}^{7}C_3 \times {}^{9}C_3 = 1764$ ✓

(c) $\frac{2}{7} \times \frac{3}{9} = \frac{2}{21}$ ✓

$1 \times 1 \times {}^{6}C_1 \times {}^{8}C_2 = 168$

Probability $= \frac{168}{1764}$

12650

16) ${}^{12}C_5 \times {}^{25}C_4 \times {}^{7}C_3 = 350\,658\,000$

(b) $1 \times 1 \times {}^{12}C_5 \times {}^{24}C_3 \times {}^{6}C_2 = 2404512$

Probability : $\frac{12}{175}$

$$\text{Probability} = \frac{{}^{12}C_5 \times {}^{25}C_3 \times {}^{6}C_2}{{}^{12}C_5 \times {}^{25}C_4 \times {}^{7}C_3}$$

$$= \frac{25!}{22!3!} \times \frac{21!4!}{25!} \times \frac{6!}{2!4!} \times \frac{4!3!}{7!}$$

$$= \frac{3}{22 \times 7}$$

$$= \frac{3}{154}$$

Challenge Exercise - Permutations, Combinations & Probability

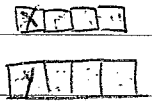
Q1

a) $\frac{5}{35} = \frac{1}{7}$ ✓

b) $\frac{20}{35} = \frac{4}{7}$ ✓

Q2

a.)



a) ${}^8C_5 \times 5! = 6720$ ✓

× (b) ${}^4P_2 \times {}^6C_3 \times 3! = 1440$ ✓

× (c) $\frac{{}^{8}\cancel{16} \times {}^6C_3 \times 3!}{6720} = \frac{\cancel{8}\,1}{7}$

3. M I S S I S S I P P I

$\frac{11!}{4! \times 4! \times 2!} = 34650$ ✓

4.

(a) $0.2 \times 0.2 = 0.04$ ✓ P(Lose) = 0.3

Win Win 2×(Win Draw) (Win Lose)×2

× (b) $0.5 \times 0.5 + (0.5 \times 0.2) + 0.5 \times 0.3$

$= 0.25 + 0.2 + 0.15 \times 2 = 0.75$

(c) $0.5 \times 0.5 = 0.25$ ✓

05. order is not important.

a. $\frac{4}{52} \times \frac{3}{51} \times \frac{2}{50} \times \frac{1}{49} \times 1 = \frac{24}{5414\,5}$?

$\frac{{}^{4}C_{4} \times {}^{48}C_{1}}{{}^{52}C_{5}} = \frac{1}{54145}$ ✓

b) $\frac{{}^{3}C_{3}}{{}^{52}C_{5}} = \frac{33}{66640}$ ✓

6.

(a) $\left(\frac{1}{2}\right)^{k} = 2^{-k}$.

complement $= 1 - \frac{1}{2^k}$ ✓

7.)

8.) $\frac{12!}{12} = 39\,916\,800$ ✓

b) $\frac{9!}{9} \times 4! = 967680$ ✓

c.) 2 friends sitting together

$\frac{11!}{11} \times 2! = 7257600$

Prob $= \frac{2}{11}$ ✓

Complement $= \frac{9}{11}$ ✓

9.) [4][3] [2][1] $4! = 24$ ✓

X (6) $^3C_3 \times 3! \times 2 = 12$ ✓ — for the person sitting not facing off

3 seats for 3 ppl → places

10. $^{30}C_8 = 5\ 852\ 925$ ✓

b) $^{10}C_5 \times {}^{10}C_2 \times {}^{10}C_1 = 113\ 400$ ✓

(c) $1 \times {}^9C_4 \times 1 \times {}^9C_1 \times {}^{10}C_1 = 11340$

Probability $= \frac{1}{10}$ ✓

11.

a) $(n-1)!$ ✓

b.) $\frac{(n-2)! \times 2!}{(n-1)!} = \frac{2}{n-1}$ ✓

c) $\frac{(n-3)!\ 3!}{(n-1)!} = \frac{6}{(n-1)(n-2)}$ ✓

d) $\frac{(n-k)!\ k!}{(n-1)!} = \frac{k!}{(n-1)(n-2)\ldots(n-k+1)}$ ✓

12.) $\frac{13+3}{52} = \frac{4}{13}$ ✓

b.) $\frac{13+3(4)}{52} = \frac{25}{52}$ ✓

(c) $\frac{13+3}{52} = \frac{4}{13}$ ✓

13.) No ✓