3 Unit Mathematics – HSC Course
Assignment 3: Trigonometry 2

Date Set: Tuesday 24th of February, 2004.
Date Due: Monday 1st of March, 2004

Question 1.

(a) Show that $\dfrac{2\sin^3 A + 2\cos^3 A}{\sin A + \cos A} = 2 - \sin 2A$, if $\sin A + \cos A \neq 0$.

(b) Prove $\dfrac{2\tan A}{1 - \tan^2 A} = \cancel{\sin 2A}\ \tan 2A$

(c) Given $\cos x = \dfrac{3}{5}$ and $\sin x$ is acute, evaluate $\sin 2x$.

(d) Simplify $\dfrac{2\tan 15^\circ}{1 - \tan^2 15^\circ}$

(e) Find the exact value of $\tan 285^\circ$

(f) Prove $\tan(45^\circ + A) - \tan(45^\circ - A) = 2\tan 2A$

Question 2.

(a) Find the exact value of $\sin\dfrac{\pi}{8}\cos\dfrac{\pi}{8}$

(b) Use $t = \tan\dfrac{\theta}{2}$ to prove $\dfrac{\cos\theta + \sin\theta - 1}{\cos\theta - \sin\theta + 1} = \tan\dfrac{\theta}{2}$

(c) Simplify $\sin\left(x + \dfrac{\pi}{4}\right)$

(d) Solve $5\cos^2\theta - 2\sin\theta\cos\theta + \sin^2\theta = 2$ for $0 \le \theta \le 360^\circ$

(e) Solve the equation $4\sin x - 2\cos x = 3$ by:

(i) the t method

(ii) the auxiliary angle method for x in the range $0 \le x \le 360^\circ$

Question 3.

(a) (i) $\int_0^{\frac{\pi}{16}} \cos^2 2x \, dx$ (ii) $\int \sin^2 3x \, dx$

(b) Find the acute angle between the lines $2x + y = 4$ and $x - y = 2$, to the nearest degree.

(c) (i) Express $\sqrt{5}\sin x + 2\cos x = -2$ in the form $R\sin(x+\alpha)$, where $R > 0$.

(ii) Hence, or otherwise, find the solutions of the equation $\sqrt{5}\sin x + 2\cos x = -2$, to the nearest minute.

(d) Find the general solution of $2\cos^2 x = 1$

(e) (i) Solve $\cos 2x = 2\sin^2 x$ for $0 \le x \le 2\pi$.

(ii) Find the exact area enclosed between the curves $y = \cos 2x$ and $y = 2\sin^2 x$ from $x = 0$ to $x = \frac{\pi}{6}$.

Question 1

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V. Good!! Check corrections!

(a) $2\dfrac{(\cancel{\sin A + \cos A})(\sin^2 A - \sin A\cos A + \cos^2 A)}{(\cancel{\sin A + \cos A})}$

$= 2(1 - \sin A\cos A)$

$= 2 - 2\sin A\cos A$

$= 2 - \sin 2A$

(b) L.H.S. $= \dfrac{2\times\frac{\sin A}{\cos A}}{1-\left(\frac{\sin A}{\cos A}\right)^2}$

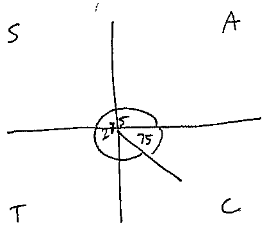
$= \dfrac{\frac{2\sin A\cos A}{\cancel{\cos^2 A}}}{\frac{\cos^2 A - \sin^2 A}{\cancel{\cos^2 A}}}$

$= \dfrac{\sin 2A}{\cos 2A}$

$= \tan 2A =$ R.H.S.

(d) $\dfrac{2\tan 15}{1-\tan^2 15} = \tan(2\times 15) = \tan 30 = \dfrac{1}{\sqrt3}$

(e)



$\tan 285 = -\tan 75 = -\tan(45+30)$

$-\left(\dfrac{\tan 45 + \tan 30}{1-\tan 45\tan 30}\right)$

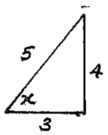
$= -\left(\dfrac{1+\frac{1}{\sqrt3}}{1-\frac{1}{\sqrt3}}\right)$

$= -\left(\dfrac{\sqrt3+1}{\sqrt3}\times\dfrac{1}{\frac{\sqrt3-1}{\sqrt3}}\right)$

$= -\left(\dfrac{\sqrt3+1}{\cancel{\sqrt3}}\times\dfrac{\cancel{\sqrt3}}{\sqrt3-1}\right)$

$= \dfrac{-\sqrt3-1}{\sqrt3-1} = \dfrac{\sqrt3+1}{1-\sqrt3}$

(c)



$\sin 2x = 2\sin x\cos x$

$= 2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$

$= \dfrac{24}{25}$

(f) $\tan(45+A) - \tan(45-A) =$

= LHS

$= \dfrac{\tan 45 + \tan A}{1-\tan 45\tan A} - \dfrac{\tan 45 - \tan A}{1+\tan 45\tan A}$

$= \dfrac{1+\tan A}{1-\tan A} - \dfrac{1-\tan A}{1+\tan A}$

$= \dfrac{(1+\tan A)^2 - (1-\tan A)^2}{1-\tan^2 A}$

$= \dfrac{(1+\tan A + 1 - \tan A)(1+\tan A - 1 + \tan A)}{1-\tan^2 A}$

$= \dfrac{2\,(2\tan A)}{1-\tan^2 A}$

$= \dfrac{4\tan A}{1-\tan^2 A} =$ LHS

$2\tan 2A = 2\left(\dfrac{2\tan A}{1-\tan^2 A}\right)$

$= \dfrac{4\tan A}{1-\tan^2 A} =$ RHS

LHS = RHS

Question 2

a) $\sin\frac{\pi}{8}\cos\frac{\pi}{8}$

$= \frac{1}{2}\left(2\sin\frac{\pi}{8}\cos\frac{\pi}{8}\right)$

$= \frac{1}{2}\left(\sin\frac{\pi}{4}\right)$ ✓

$= \frac{1}{2}\times\frac{1}{\sqrt{2}} = \frac{1}{2\sqrt{2}}$ ✓

b) $\dfrac{\cos\theta+\sin\theta-1}{\cos\theta-\sin\theta+1} = \dfrac{\frac{1-t^2}{1+t^2}+\frac{2t}{1+t^2}-1}{\frac{1-t^2}{1+t^2}-\frac{2t}{1+t^2}+1}$ ✓

$= \dfrac{\frac{1-t^2}{1+t^2}+\frac{2t}{1+t^2}-\frac{1+t^2}{1+t^2}}{\frac{1-t^2}{1+t^2}-\frac{2t}{1+t^2}+\frac{1+t^2}{1+t^2}}$ ✓

$= \dfrac{\cancel{1}-t^2+2t-\cancel{1}-t^2}{\cancel{1}-\cancel{t^2}-2t+\cancel{1}+\cancel{t^2}}$ ✓

$= \dfrac{2t-2t^2}{2-2t}$ ✓

$= \dfrac{2t\cancel{(1-t)}}{2\cancel{(1-t)}} = t = \tan\dfrac{\theta}{2}$

c) $\sin x\cos\frac{\pi}{4} + \cos x\sin\frac{\pi}{4}$

$= \sin x\times\frac{1}{\sqrt{2}} + \cos x\times\frac{1}{\sqrt{2}}$ ✓

$= \dfrac{\sin x+\cos x}{\sqrt{2}}$ ✓

d) $\cancel{5\cos^2\theta - 2\sin\theta\cos\theta + \sin^2\theta = 2}$

$\cancel{4\cos^2\theta - 2\sin\theta\cos\theta + 1 = 2}$

$\cancel{4\cos^2\theta - 2\sin\theta\cos\theta - 1 = 0}$

$\cancel{\text{let } 2\cos\theta = u}$

$\cancel{u^2 - \sin\theta\, u}$

$\cancel{4\cos^2\theta - 2\sin\theta\cos\theta - \sin^2\theta - \cos^2\theta = 0}$

$\cancel{3\cos^2\theta - 2\sin\theta\cos\theta - \sin^2\theta = 0}$

$\cancel{4\cos^2\theta - 1 = \sin 2\theta}$

$\cancel{(2\cos\theta+1)(2\cos\theta-1) =}$

$5\cos^2\theta - 2\sin\theta\cos\theta + \sin^2\theta = 2$

$1 - 2\sin\theta\cos\theta = 2 - 4\cos^2\theta$

$\sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta = 2(\sin^2\theta+\cos^2\theta) - 4\cos^2\theta$

$(\sin\theta-\cos\theta)^2 = 2\sin^2\theta - 2\cos^2\theta$

$(\sin\theta-\cos\theta)^2 = 2(\sin\theta+\cos\theta)(\sin\theta-\cos\theta)$

$\sin\theta-\cos\theta = 2(\sin\theta+\cos\theta)$

$\sin\theta-\cos\theta = 2\sin\theta+2\cos\theta$

$-\sin\theta = 3\cos\theta$

$\sin^2\theta = 9\cos^2\theta$

Do not square both sides unless $\sqrt{\ }$ is involved

Note: $\div$ by $\cos\theta$

$\tan\theta = -3 \Rightarrow \theta = 108°26'$ or $288°26'$

$\sin^2\theta + \cos^2\theta = 10\cos^2\theta$

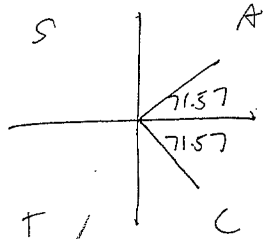
$1 = 10\cos^2\theta$

$0.1 = \cos^2\theta$

$\sqrt{0.1} = \cos\theta$

$\theta \doteqdot \cancel{71.57°}$ (to 2 dp), or $288.43°$ (to 2dp)

$\cancel{108°26'}$



e) i) $4\sin x - 2\cos x = 3$

Let $t = \tan\frac{x}{2}$

$$\frac{4(2t)}{1+t^2} - \frac{2(1-t^2)}{1+t^2} = 3$$

$$= \frac{8t - (2-2t^2)}{1+t^2} = 3$$

$$= \frac{8t - 2 + 2t^2}{1+t^2} = 3$$

$$2t^2 + 8t - 2 = 3t^2 + 3$$

$$0 = t^2 - 8t + 5$$

$$t = \frac{8 \pm \sqrt{64-20}}{2}$$

$$= 4 \pm \sqrt{11}$$

Do not forget to test for 180° when using 't' result.

$\frac{x}{2} = 82.22\ldots$ or $34.347\ldots$

$x \doteqdot 164.43°$ (to 2dp) or $68.70°$ (to 2dp)

ii) ~~$4\sin x - 2\cos x = 3$~~

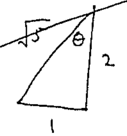
~~$R = \sqrt{4^2 + 2^2}$~~

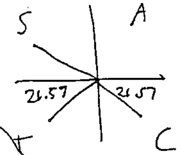
~~$= \sqrt{20}$~~

~~$= 2\sqrt{5}$~~

~~$\alpha = \tan^{-1} \frac{-2}{4}$~~

~~$= \tan^{-1} -\frac{1}{2} = 153.43°$ or $333.43°$ (to 2dp)~~





ii) $R = \sqrt{4^2+2^2}$

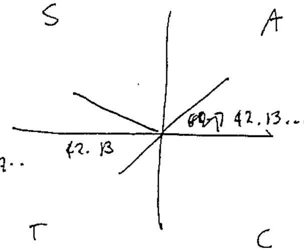
$= \sqrt{20}$

$= 2\sqrt{5}$

$\tan^{-1}\frac{+2}{4} = \alpha = +26.57°$ (to 2dp)

$2\sqrt{5}\sin(x - 26.57°) = 3$

$\sin(x - 26.57°) = \frac{3}{2\sqrt{5}}$



$x - 26.57 = 42.1304\ldots$ or $137.8695\ldots$

$x = 68.70°$ or $164.43°$ (to 2dp)

Question 3

a) i) $\displaystyle\int_0^{\frac{\pi}{16}} \cos^2 2x \, dx$

$= \left[(2\cos 2x)\left(\frac{1}{2}\sin 2x\right)\right]_0^{\frac{\pi}{16}}$

$= 0.35355339\ldots - 2\left(\frac{1}{2}\times 0\right)$

$= 0.35355339\ldots$

Use $\cos 2\alpha = 2\cos^2\alpha - 1$ and substitute $\alpha = 2x$

$\therefore \cos 4x = 2\cos^2 2x - 1$

$\therefore \cos^2 2x = \frac{1}{2}(\cos 4x + 1)$

$\displaystyle\int_0^{\frac{\pi}{16}} \cos^2 2x \, dx$

$= \displaystyle\frac{1}{2}\int_0^{\frac{\pi}{16}} \cos 4x + 1 \, dx \ldots$ continue.

ii) $\displaystyle\int \sin^2 3x \, dx$

$= (2\sin 3x)\left(-\frac{1}{3}\cos 3x\right)$

Similarly $\sin^2 3x = \dfrac{1-\cos 6x}{2}$

$\displaystyle\frac{1}{2}\int 1 - \cos 6x \, dx \ldots$ continue.

c) i) $R = \sqrt{(\sqrt{5})^2 + 2^2}$

$= \sqrt{9}$

$= 3$ ✓

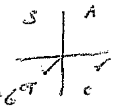
$\alpha = \tan^{-1}\dfrac{\sqrt{5}}{2} \doteqdot 57.69°$ ✓ (to 2dp)

$\sqrt{5}\sin x + 2\cos x = 3\sin(x + 57.69°)$

ii) $-2 = 3\sin(x + 57.69°)$ for $0 \le x \le 360°$

$\dfrac{-2}{3} = \sin(x + 57.69°)$

$57.69° \le x + 57.69° \le 417.69°$



$\sin^{-1}\dfrac{-2}{3} = 41.81 = x + 57.69$

(to 2 dp)

$x + 57.69 = 221.14°$ or $318.36°$

$x = 163°27'$ or $261°10'$

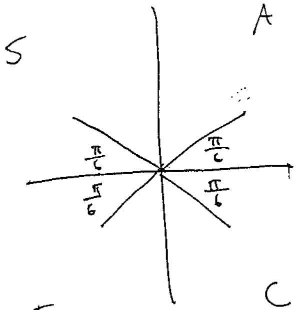
$x = 15.9°$ (to 1dp) $= 15°53'$ (to nearest minute)

X X

d) $\cos^2 x = \dfrac{1}{2}$

$\cos x = \pm\dfrac{1}{\sqrt{2}}$ general solⁿ.

$x = \dfrac{\pi}{4}$



e) i) $\cos 2x = 2\sin^2 x$

$\cos^2 x - \sin^2 x = 2\sin^2 x$

$\cos^2 x = 3\sin^2 x$ or $\Rightarrow \tan^2 x = \frac{1}{3}$

$\tan x = \pm\frac{1}{\sqrt{3}}$ ... try again

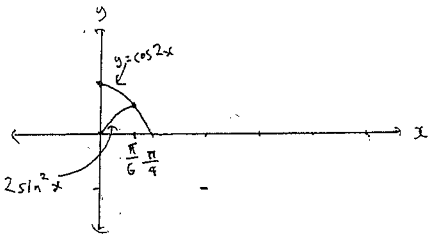
$4\cos^2 x = 3$ ✓

$\cos^2 x = \dfrac{3}{4}$

$\cos x = \pm\dfrac{\sqrt{3}}{2}$ ✓

$x = \dfrac{\pi}{6},\ 2\pi - \dfrac{\pi}{6} = \dfrac{11\pi}{6}$ ✓, $\pi - \dfrac{\pi}{6} = \dfrac{5\pi}{6},\ \pi + \dfrac{\pi}{6} = \dfrac{7\pi}{6}$ ✓

i.)



$2(\sin x)^2$

$$\int_0^{\frac{\pi}{6}} \cos 2x - 2\sin^2 x \; dx$$

use $\cos 2x = 1 - 2\sin^2 x$

$\therefore 2\sin^2 x = 1 - \cos 2x$ .. try again

$$= \left[\frac{1}{2}\sin 2x - \cancel{4\sin^2 x(-\cos x)}\right]_0^{\frac{\pi}{6}}$$

$$= \left[\frac{\sin 2x}{2} + 4\sin^2 x \cos x\right]_0^{\frac{\pi}{6}}$$

$$= \left(\frac{\sqrt{3}}{4} + 4 \times \frac{1}{4} \times \frac{\sqrt{3}}{2}\right) - (0 + 0)$$

$$= \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{2}$$

$$= \frac{3\sqrt{3}}{4} \text{ units}^2$$