CHALLENGE EXERCISE 12

1. Write $P(x) = x^5 + 2x^4 + x^3 - x^2 - 2x - 1$ as a product of its factors.
2. A polynomial $P(x) = (x - b)^7 Q(x)$.
   (a) Show that $P(b) = P'(b) = 0$.
   (b) Hence find a and b, if $(x - 1)^7$ is a factor of
   $P(x) = x^7 + 3x^6 + ax^5 + x^4 + 3x^3 + bx^2 - x - 1$.
3. Solve $\tan^4\theta - \tan^3\theta - 3\tan^2\theta + 3\tan\theta = 0$ for $0° \leq \theta \leq 360°$.
4. (a) Find the equation of the tangent to the curve $y = x^3$ at the point where $x = -1$.
   (b) Find the point where this tangent cuts the curve again.
5. (a) Find the remainder when $p(x) = 2x^4 - 7x^3 + ax^2 + 3x - 9$ is divided by $2x - 1$.
   (b) If the remainder, when $p(x)$ is divided by $x + 2$, is 17, find the value of a.
6. If α, β and γ are roots of the cubic equation $2x^3 + 8x^2 - x + 6 = 0$, find
   (a) $\alpha\beta\gamma$
   (b) $\alpha^2 + \beta^2 + \gamma^2$
7. Solve $4\sin^3\theta - 3\sin\theta - 1 = 0$ for $0° \leq \theta \leq 360°$.
8. Find the value of a if $(x + 1)(x - 2)$ is a factor of $2x^3 - x^2 + ax - 2$.
9. Prove that if $x - a$ is a factor of polynomial $P(x)$, then $P(a) = 0$.

CHALLENGE EXERCISE 12

1. $P(x) = (x - 1)(x + 1)^2(x^2 + x + 1)$

2. (a)
$$\begin{aligned} P(b) &= (b - b)^7 Q(b) \\ &= 0 \\ P'(x) &= (x - b)^7 Q'(x) \\ &\quad + Q(x)\,7(x - b)^6 \\ P'(b) &= (b - b)^7 Q'(b) \\ &\quad + Q(b)\,7(b - b)^6 \\ &= 0 \end{aligned}$$

(b) $a = -8\frac{1}{3}$, $b = 2\frac{1}{3}$

3. $\theta = 0°, 45°, 60°, 120°, 180°, 225°, 240°, 300°, 360°$ 4. (a) $3x - y + 2 = 0$ (b) $(2, 8)$

5. (a) $\frac{a - 33}{4}$ (b) $a = -14$ 6. (a) -3 (b) 17

7. $\theta = 90°, 210°, 330°$ 8. $a = -5$

9. If $x - a$ is a factor of $P(x)$
Then $P(x) = (x - a)\,Q(x)$
$\therefore P(a) = (a - a)\,Q(a)$
$= 0$