Applications of Differentiation

ASSIGNMENT

Exercises

- 1. Find two numbers which added together give 24 and the sum of their squares is a minimum.
- 2. A length of fencing of 2500 metres is to enclose a rectangular paddock. Find the dimensions of the paddock such that the area enclosed is a maximum.
- 3. The perimeter of a rectangle is 20 cm. Find the dimensions of the rectangle such that the square on its diagonal is a minimum.

4.

A rectangle of side 'x' cm, as shown in the diagram is drawn inside a right-angled isosceles triangle of height 8 cm.

Find the dimensions of the shaded part so that its area is a maximum.

5. A rectangular box has a square base of side 'x' cm and no top.

The volume is 108 c.cm.

Find the value of 'x' so that the surface area of the box is a minimum.

- 6. A straight line passing through the point (2,3) cuts the positive direction of the x and y axes at (p,o) and (o,q) respectively.
 - (i) Show that $\frac{2}{p} + \frac{3}{q} = 1$.
 - (ii) Show that $A = \frac{q^2}{q-3}$ where A is the area of the triangle formed by the line and the co-ordinate axes.
 - (iii) Find the minimum value of A.
- 7. The surface area of a box in the shape of a rectangular prism, is 300 sq cm. If the box is to have a square base, find the dimensions of the box that will make the volume a maximum, when it is open at the top.
- 8. A square piece of cardboard of side 6 cm has squares of x cm cut out of each corner and the cardboard then folded to make a box.
 - (i) Find V, the volume of the box, in terms of x.
 - (ii) Find the value of \dot{x} for which this volume is a maximum.
 - (iii) Find this maximum volume.

- A spherical balloon is inflated so that its volume increases at the rate of 2 cubic centimetres per minute.
 - At what rate is the radius changing at the instant it is 5 centimetres.
- 10. Show that the rate of change of the area of a circle with respect to its radius, is equal to its circumference.
- 11. Show that the rate of change of the volume of a sphere with respect to its radius is equal to its surface area.
- 12. Show that the rate of change of the area of a square with respect to its side is equal to half of its perimeter.
- 13. For what positive value of x does the function $y = \sqrt{x} + \frac{4}{\sqrt{x}}$ have a minimum value.
- 14. The sum of the base and perpendicular height of a triangle is 10 cm.

 If the base is x cm, find the dimensions of the perpendicular height and the base which will make the area of the triangle a maximum.
- 1. 12 and 12
- 2. A square of side 625 metres
- 3. A square of side 5 cm
- 4. A square of side 4 cm
- 5. x = 6 cm
- 6. 12 units of area
- 7. $10 \text{ cm} \times 10 \text{ cm} \times 5 \text{ cm}$
- 8. (i) $V = x(6-2x)^2$
 - (ii) x = 1
 - (iii) 16 c.cm
- 9. $\frac{1}{50\pi}$ cm/min
- 13. x = 4
- 14. base = height = 5 cm