

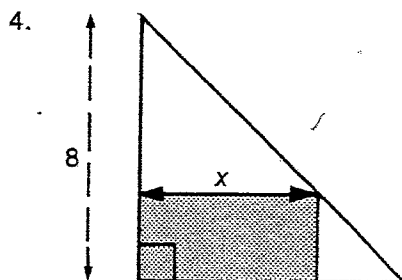
Applications of Differentiation

ASSIGNMENT

5

Exercises

- Find two numbers which added together give 24 and the sum of their squares is a minimum.
- A length of fencing of 2500 metres is to enclose a rectangular paddock. Find the dimensions of the paddock such that the area enclosed is a maximum.
- The perimeter of a rectangle is 20 cm. Find the dimensions of the rectangle such that the square on its diagonal is a minimum.



A rectangle of side 'x' cm, as shown in the diagram is drawn inside a right-angled isosceles triangle of height 8 cm.

Find the dimensions of the shaded part so that its area is a maximum.

- A rectangular box has a square base of side 'x' cm and no top. The volume is 108 c cm. Find the value of 'x' so that the surface area of the box is a minimum.
- A straight line passing through the point (2,3) cuts the positive direction of the x and y axes at (p,0) and (0,q) respectively.
 - Show that $\frac{2}{p} + \frac{3}{q} = 1$.
 - Show that $A = \frac{q^2}{q-3}$ where A is the area of the triangle formed by the line and the co-ordinate axes.
 - Find the minimum value of A.
- The surface area of a box in the shape of a rectangular prism, is 300 sq cm. If the box is to have a square base, find the dimensions of the box that will make the volume a maximum, when it is open at the top.
- A square piece of cardboard of side 6 cm has squares of x cm cut out of each corner and the cardboard then folded to make a box.
 - Find V, the volume of the box, in terms of x.
 - Find the value of x for which this volume is a maximum.
 - Find this maximum volume.

9. A spherical balloon is inflated so that its volume increases at the rate of 2 cubic centimetres per minute.
At what rate is the radius changing at the instant it is 5 centimetres.
10. Show that the rate of change of the area of a circle with respect to its radius, is equal to its circumference.
11. Show that the rate of change of the volume of a sphere with respect to its radius is equal to its surface area.
12. Show that the rate of change of the area of a square with respect to its side is equal to half of its perimeter.
13. For what positive value of x does the function $y = \sqrt{x} + \frac{4}{\sqrt{x}}$ have a minimum value.
14. The sum of the base and perpendicular height of a triangle is 10 cm.
If the base is x cm, find the dimensions of the perpendicular height and the base which will make the area of the triangle a maximum.
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1. 12 and 12
2. A square of side 625 metres
3. A square of side 5 cm
4. A square of side 4 cm
5. $x = 6$ cm
6. 12 units of area
7. 10 cm \times 10 cm \times 5 cm
8. (i) $V = x(6 - 2x)^2$
(ii) $x = 1$
(iii) 16 c.cm
9. $\frac{1}{50\pi}$ cm/min
13. $x = 4$
14. base = height = 5 cm