BINOMIAL THEOREM

1) Write the 1st 3 terms of each of the following expansions:

   a) $(x+3)^6$ b) $(x-y)^4$ c) $(x-y^2)^3$ d) $(x-\frac{1}{x})^5$

2) Find the coefficient of x^3 in $(x+2)^5$

3) Write down the:

   a) 6th term of the expansion of $(x+y)^{10}$

   b) 4th term of the expansion $(x-2y)^8$

   c) 5th term of the expansion $(4x-3y)^9$

   d) constant term in the expansion of i) $(x+\frac{1}{x})^6$

                                     ii) $(2x+\frac{1}{x^2})^{12}$

   e) independent of x in $(x-\frac{1}{x^2})^9$

   f) 3rd term in the expansion of $\displaystyle\sum_{k=0}^{6} \binom{6}{k} x^{6-k} 2^k$

4) Write in sigma notation the expansion of:

   a) $(x+2y)^5$ b) $(x-\frac{3}{x})^8$

*5) Use binomial expansion to find $(2.02)^{10}$ correct to 2 decimal places. $(2+0.2)$

6) Prove $^7C_{k+2} = \, ^6C_{k+1} + \, ^6C_{k+2}$

7) Simplify: a) $^8C_k \div \, ^8C_{k-1}$

           b) $\binom{12}{k} \div \binom{12}{k-2}$

8) Find the greatest coefficient in the expansion of $(4+5x)^{10}$

9) Find the largest term in the expansion of $(1-2x)^9$ when $x = \frac{1}{3}$.

*10) Find the greatest coefficient in the expansion of $\left(x + \frac{1}{x}\right)^6$

11) Find the middle term(s) in the expansion of:

   a) $\left(x^2 - \frac{1}{2x}\right)^{10}$ b) $\left(3a + \frac{x^3}{6}\right)^9$

12) Use the identity $(1+x)^4(1+x)^3 = (1+x)^7$ to prove that: $\binom{4}{2} + \binom{4}{1}\binom{3}{1} + \binom{3}{2} = \binom{7}{2}$

13) If $(1+x)^n = \sum_{k=0}^{n} \binom{n}{k}x^k$, show that:

$$\sum_{k=0}^{n} 3^k \binom{n}{k} = 4^n$$

*14) Prove: a) $\sum_{k=1}^{n} \binom{n}{k} = 2^n - 1$

   b) $\sum_{k=0}^{n} \binom{n}{k}(-2)^k = (-1)^n$

   c) $\sum_{k=0}^{n} 3^k \binom{n}{k} = 2^{2n}$

   d) $\binom{n}{0} + \frac{1}{2}\binom{n}{1} + \frac{1}{3}\binom{n}{2} + \cdots + \frac{1}{n+1}\binom{n}{n} = \frac{2^{n+1} - 1}{n+1}$

#15) Three consecutive coefficients in the expansion of $(1+x)^n$ are in the ratio $6:3:1$. Find the value of n.

*16) Find n if the coefficients of the 2nd, 3rd and 4th terms in the expansion of $(1+x)^n$ form successive terms of an arithmetic sequence.

2.

ANSWERS

1) a) $x^6 + 18x^5 + 135x^4$ c) $x^3 - 3x^2y^2 + 3xy^4$

   b) $x^4 - 4x^3y + 6x^2y^2$ d) $x^5 - 5x^3 + 10x$

2) 40

3) a) $252x^5y^5$ d) i) 20   ii) ~~495~~   126 720

   b) $-448x^5y^3$ e) ~~~~184

   c) $10\,450\,944\,x^5y^4$ f) $60x^4$

4) a) $\displaystyle\sum_{k=0}^{5} {}^5C_k\, x^{5-k}(2y)^k$ b) $\displaystyle\sum_{k=0}^{8} {}^8C_k\, x^{8-k}\left(-3/x\right)^k$

5) $1131 \cdot 13$

6) Proof req'd

7) a) $\dfrac{9-k}{k}$ b) $\dfrac{(14-k)(13-k)}{k(k-1)}$

8) $8 \cdot 4 \times 10^8$

9) $T_7 = 5376$ 10) 20

11) a) $T_6 = \dfrac{63x^5}{8}$ b) $T_5 = \dfrac{189}{8}\, a^5 b^{12}$

   and $T_6 = \dfrac{21}{16}\, a^4 b^{15}$

12) 13) 14) Proofs req'd.

15) 11

16) 7

3.