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BRIGIDINE COLLEGE RANDWICK

3 Unit Mathematics **5 June 2003**

Calculus and Inverse Trigonometry **Time: 45 min**

All questions may be attempted.
Neatness may be taken into consideration in the awarding of marks.

1. Given that $f(x) = x^2 - 2x + 3$

   a. Show that f(x) may be rewritten as $f(x) = (x - 1)^2 + 2$ and state its Domain and Range. 2 m

   b. By considering the positive real numbers such that f(x) is a one-to-one function, determine the equation of $f^{-1}(x)$. 2 m

2. Determine the Exact Value of $\sin\left(\cos^{-1}\left(\frac{-2}{3}\right)\right)$ 3 m

3. Consider $f(x) = \sin^{-1}(x + 1)$

   a. State the values of $\boxed{x}$ for which f(x) is defined. 1 m

   b. Sketch f(x). 3 m

PM

4. 0 is a fixed point on a given straight line. A Particle moves along this line and its displacement x cm, from 0 at any given time, t seconds, after its start of motion is given by $x = 2 + \cos 2t$.

   a. By finding an expression for the acceleration, explain why this motion represents Simple Harmonic Motion and state the centre of its motion. 3 m

   b. State the first two occasions when the particle is at rest and the displacements on these two occasions. 3 m

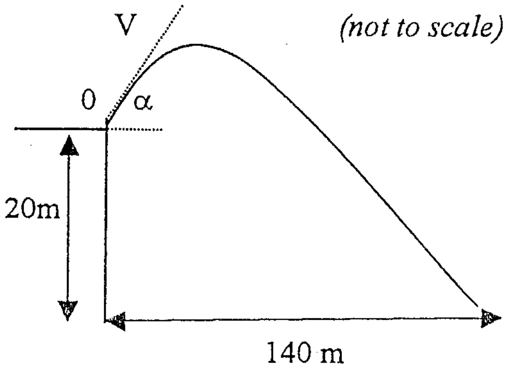
   c. State the amplitude and period of the motion. 2 m

5. a. Prove that $\dfrac{d^2x}{dt^2} = \dfrac{d(\frac{1}{2}v^2)}{dx}$ 1m

   b. If the acceleration of a particle is $2x - 3x^2$ and $v = -2$ when $x = 0$, find v in terms of x. 2 m

- please turn over -

6.



A stone is thrown from a point 0 at the top of a cliff 20 metres above a beach.

The stone is thrown at an angle of elevation α above the horizontal and with a speed of 35 m/s.

The stone hits the beach at a point which is 140 m (horizontally) from the cliff.

a. Show that the horizontal component of the velocity may be expressed as $35 \cos \alpha$. 1 m

b. Consider this point 0 of projection as (0,0) and take gravity as $g = 10$ m/s^2. Show that the parametric equations of motion may be given by: 2 m

$$x = 35t \cos \alpha \qquad \text{and} \qquad y = -5t^2 + 35t \sin \alpha .$$

c. Show that $\tan \alpha = \dfrac{3}{4}$ or $\tan \alpha = 1$. 2 m

d. Hence find the two possible times for which the stone is in the air, giving answers in exact form. 2 m

7. Newton's Law of cooling T may be given by the formula $T = P + Ae^{-kt}$, where P represents the Temperature of the surroundings, A the initial temperature of the body and t the time in hours.

a. Show that the rate at which a body loses heat is proportional to the difference between the temperature of the body and the temperature of the surrounding air. 1 m

At 1 am, Forensic scientists investigating a murder found that the temperature of a body was 22°C when it was first measured. Two hours later the temperature of the body had fallen to 19.5 °C. The room was at a constant temperature of 19 °C.

b. If normal body temperature is 37 °C, show that A = 3 °C. 1 m

c. Determine the time of death. 3 m

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1) a) $f(x) = x^2 - 2x + 1 + 2$

$f(x) = (x-1)^2 + 2$

D: Reals

R: $y \geq 2$

2

b) $\therefore x \geq 1$

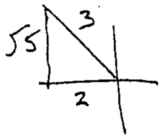
$x = (y-1)^2 + 2$

$\pm\sqrt{x-2} = y - 1$

$y = \sqrt{x-2} + 1$

1

2) $\sin\left(\cos^{-1}\left(-\frac{2}{3}\right)\right)$



$\cos^{-1}\left(-\frac{2}{3}\right) = x$

$-\frac{2}{3} = \cos x$

$0 \leq x \leq \pi$ 2nd quad

3

$\therefore \sin x = \frac{\sqrt{5}}{3}$

3) $f(x) = \sin^{-1}(x+1)$

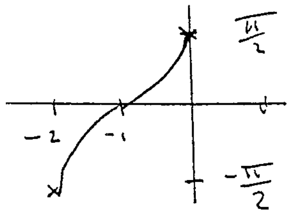
$-1 \leq x+1 \leq 1$

1

$-2 \leq x \leq 0$

$x = -2$ $\sin^{-1}(-1) = -\frac{\pi}{2}$

$x = 0$ $\sin^{-1}(1) = \frac{\pi}{2}$



Ends curvature

3

4) a) $x = 2 + \cos 2t$

$\dot{x} = -2\sin 2t$

$\ddot{x} = -4\cos 2t$

1

$= -4[x-2]$

1

Centre $x = 2$

Since from

$\ddot{x} = -n \boxed{}$

1

$\therefore$ influenced by force & is proportional to the distance from $x = c$

b) At rest $\dot{x} = 0$

$-2\sin 2t = 0$

$2t = 0, \pi, 2\pi, \ldots$

$t = 0, \frac{\pi}{2}, \pi$

3

$\therefore t = \frac{\pi}{2}$

$x = 2 + \cos\pi = 1$

$t = \pi$

$x = 2 + \cos 2\pi = 3$

c) $x = 2 + \cos 2t$

$a = 1$ $T = \frac{2\pi}{2}$

2

$= \pi$ seconds

5a) $\dot{x} = \frac{dv}{dt}$

1

$= \frac{dv}{dx}\frac{dx}{dt}$

$= v\frac{dv}{dx}$

$= \frac{d\frac{1}{2}v^2}{dv}\frac{dv}{dx} = \frac{d(\frac{1}{2}v^2)}{dx}$

b) $\frac{d\frac{1}{2}v^2}{dx} = 2x - 3x^2$

$\frac{1}{2}v^2 = x^2 - x^3 + c$

$v = 2$, $x = 0$: $2 = c$

2

$v^2 = 2x^2 - 2x^3 + 4$

$v = -\sqrt{2x^2 - 2x^3 + 4}$

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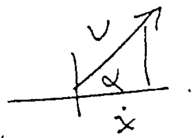
6/ a/ $\cos\alpha = \frac{\dot{x}}{v}$



$\therefore \dot{x} = v\cos\alpha$ 1

b/ $x = 0$

$\dot{x} = v\cos\alpha$

$x = v\cos\alpha t + c$

$t = 0 \quad x = 0$

$x = 35\, t \cos\alpha$

$\ddot{y} = -10$

$\dot{y} = -10t + c$

$\dot{y} = -5t^2 + \ldots v\sin\theta$ 2

$y = -5t^2 + 35\sin\alpha t + c \Rightarrow \begin{matrix} t=0 \\ c=0 \end{matrix}$

$y = -5t^2 + 35\, t \sin\alpha$

c/ NB

$y = x\tan\alpha - \frac{5x^2}{35^2}(1+\tan^2\alpha)$

$(140, -20)$

$-20 = 140\tan\alpha - 80 - 80\tan^2\alpha$

$\Rightarrow 80\tan^2\alpha - 140\tan\alpha + 60 = 0$ 2

$20[4\tan^2\alpha - 7\tan\alpha + 3] = 0$

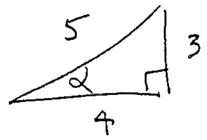
$[4\tan\alpha - 3][\tan\alpha - 1] = 0$

$\tan\alpha = \frac{3}{4} \;\Big|\; \tan\alpha = 1$

or

from b. simultaneous

d/ $\tan\alpha = \frac{3}{4}$



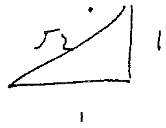
$\cos\alpha = \frac{4}{5}$

$x = 35\, t\cos\alpha$

$140 = 35\, t\, \frac{4}{5}$

$5s = t$

$\tan\alpha = 1$



$\cos\alpha = \frac{1}{\sqrt{2}}$ 2

$140 = 35\, t\, \frac{1}{\sqrt{2}}$

$4\sqrt{2}\, s = t$

7/ $T = P + Ae^{-kt}$

a/ $\frac{dT}{dt} = -kAe^{-kt}$

$= -k[T - P]$ 1

$\therefore$ rate $\propto$ [diff]

b/ Let 1 Am $t = 0$

since measure temp

$\therefore T = P + Ae^{-kt}$

$22 = 19 + A \quad \boxed{A = 3}$ 1

$t = 2$

$T = 19 + 3e^{-kt}$

$19.5 = 19 + 3e^{-2k}$

$\frac{1}{2} = 3e^{-2k}$

$-\frac{1}{2}\ln\frac{1}{6} = k$

$37 = 19 + 3e^{\frac{1}{2}\ln\frac{1}{6} t}$

$6 = e^{\square}$

$\frac{2\ln 6}{\ln 1/6} = t$ 3

$-2 = t$

$\therefore$ 1 Am − 2 hrs

11 pm