BRIGIDINE COLLEGE RANDWICK

MATHEMATICS | **15 June 2006**

Extension 1 Paper | **Time 45 min**

Write your name at the top of this exam. 15
Neatness may be taken into consideration in the awarding of marks.

There are 7 questions.

1. Evaluate $\int_0^1 \frac{x}{\sqrt{1 + x}}\, dx$ as a single expression and by using the substitution $u = 1 + x$. *3 m*

2\. a. Show that acceleration may be given by $\frac{d\left(\frac{1}{2}v^2\right)}{dx}$. 0 *1 m*

b. The acceleration of a particle of mass 5 kg moving in a straight line is given by $\frac{7 - 2x}{x^3}$. The particle starts from rest at a point $x = \frac{1}{4}$ from 0. 2 *3 m*

Show that its speed in terms of x may be give by

$$v = \frac{1}{x}\sqrt{96x^2 + 4x - 7}.$$

3. N is the number of animals in a certain population at time t years. The population size N satisfies the equation $\frac{dN}{dt} = -k(N - 1000)$, for some constant k.

a. Verify by differentiation that $N = 1000 + Ae^{-kt}$, A constant, is a solution of the equation. 1 *1 m*

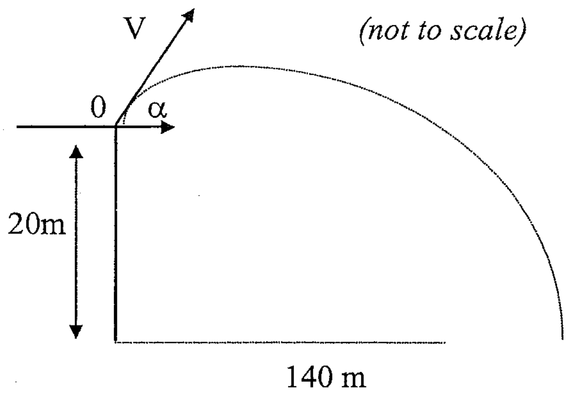
b. Initially there are 2500 animals but after 2 years there are only 2200 left. Find the values of A and K. 2 *2 m*

c. Find when the number of animals has fallen to 1300. (nearest half-year) 2 *2 m*

generous!

- please turn over -

4.



A stone is thrown from a point 0 at the top of a cliff 20 metres above a beach.

The stone is thrown at an angle of elevation α above the horizontal and with a speed of 35 m/s.

The stone hits the beach at a point which is 140 m (horizontally) from the cliff.

a. Consider this point 0 of projection as (0,0) and take gravity as $g = 10$ m/s^2. Show that the parametric equations of motion may be given by: 2

$$x = 35t\cos\alpha \quad \text{and} \quad y = -5t^2 + 35t\sin\alpha .$$ *2 m*

b. Show that $\tan\alpha = \dfrac{3}{4}$ or $\tan\alpha = 1$. 3 *3 m*

c. Hence find the two possible times for which the stone is in the air, giving answers in exact form. 2 *2 m*

5. The rise and fall of the tide at a certain harbour may be considered as Simple Harmonic Motion. The interval of time between successive high tides is approximately $12\frac{1}{2}$ hours. 1 *4 m*

The depth of the water at a certain point in the harbour is 6 metres at low tide and 14 metres at high tide. If low tide occurs at noon, at what time is the depth of the water in this harbour 12 metres?

6. Evaluate $\sin\left(2\cos^{-1}\frac{8}{17}\right)$ and express your answer as a rational number. 1 *3 m*

7. Consider $f(x) = \cos^{-1}(2x + 1)$

a. State the values of x for which f(x) is defined. 1 *1 m*

b. Sketch f(x). 2 *2 m*

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4/ a) hor

$\ddot{x} = 0$

$\dot{x} = v\cos\alpha$

$x = v\cos\alpha\, t + c$

$t = 0,\ x = 0$

$x = 35t\cos\alpha$

Vert

$\ddot{y} = -10$ $\qquad t = 0,\ \dot{y} = v\sin\alpha$

$\dot{y} = -10t + c$

$\dot{y} = -10t + v\sin\alpha$

$y = -5t^2 + v\sin\alpha\, t + c$

$y = -5t^2 + v\sin\alpha\, t$

$\therefore y = -5t^2 + 35t\sin\alpha$ 2

b) $y = x\tan\alpha - \frac{5x^2}{35^2}(1 + \tan^2\alpha)$

$(140, -20)$

$-20 = 140\tan\alpha - 80 - 80\tan^2\alpha$

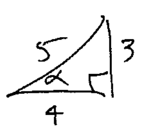
$80\tan^2\alpha - 140\tan\alpha + 60 = 0$

$20[4\tan^2\alpha - 7\tan\alpha + 3] = 0$

$20[4\tan\alpha - 3][\tan\alpha - 1] = 0$

$\therefore \tan\alpha = \frac{3}{4}$ | $\tan\alpha = 1$ 3

c) $\tan\alpha = \frac{3}{4}$



$x = 35t\cos\alpha$

$140 = 35t\ \frac{4}{5}$

$t = 5s$ 2

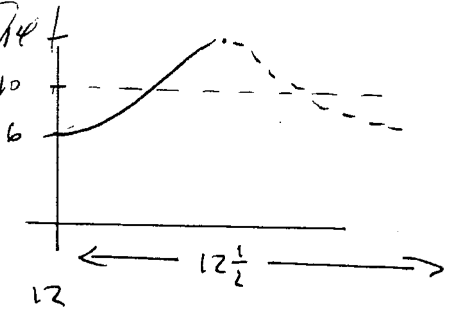
$\tan\alpha = 1$

$\cos\alpha = \frac{1}{\sqrt{2}}$

$140 = 35t\frac{1}{\sqrt{2}}$

$t = 4\sqrt{2}s$

5/



$n = \frac{2\pi}{25/2}$

$n = \frac{4\pi}{25}$

$x = -4\cos\frac{4\pi}{25}t + 10$

$x = 12$

$12 = -4\cos\frac{4\pi}{25}t + 10$

$-\frac{1}{2} = \cos\frac{4\pi}{25}t$

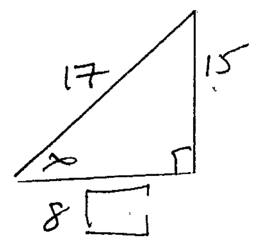
$\frac{2\pi}{3} = \frac{4\pi}{25}t$

$t = 4\frac{1}{6}$ $\therefore$ 4:10 pm 4

6/ $\sin\left[2\cos^{-1}\frac{8}{17}\right]$

$x = \cos^{-1}\frac{8}{17}$

$\cos x = 8/17$



$\sin 2x$

$= 2\sin x\cos x$

$= 2\ \frac{8}{17}\ \frac{15}{17} = \frac{240}{289}$ 3

7) a) $-1 \le 2x+1 \le 1$

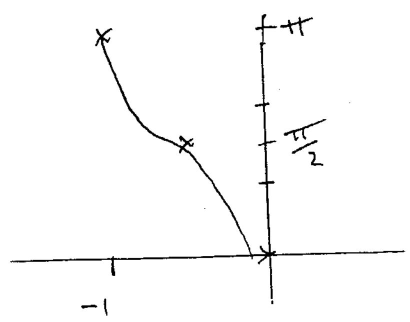
$-2 \le 2x \le 0$

$-1 \le x \le 0$ 1

b) $f(x) = \cos^{-1}(2x+1)$

$x = -1$ $\quad x = -\frac{1}{2}$ $\quad x = 0$

$y = \pi$ $\quad y = \frac{\pi}{2}$ $\quad 0$



2

Ext1 Yr12 15/6/6

1) $\int_0^1 \frac{x}{\sqrt{1+x}}\,dx$ $\quad u = 1+x,\ du = dx,\ x = u-1$

$= \int_{1}^{2} (u-1)u^{-1/2}\,du$ ✓

$= \int (u^{1/2} - u^{-1/2})\,du$ $\quad$ ~~N3~~

$= \frac{2}{3}u^{3/2} - 2u^{1/2} \Big|_1^2$

$= \left(\frac{2}{3}(2\sqrt{2}) - 2\sqrt{2}\right) - \left(\frac{2}{3} - 2\right)$

$= \frac{-2\sqrt{2}}{3} + \frac{4}{3}$ ✓ $\boxed{\int_1^2 \text{wrong fn'}}$

$= \frac{4 - 2\sqrt{2}}{3}$ ✓ $\quad$ 3

2) a) $\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt}$

$= \frac{dv}{dx} v$ $\quad$ 1

$= \frac{dv}{dx} \frac{d\frac{1}{2}v^2}{dv}$

$= \frac{d\frac{1}{2}v^2}{dx}$ ✓

b) $\frac{d\frac{1}{2}v^2}{dx} = \frac{7-2x}{x^3}$

$\frac{1}{2}v^2 = \int 7x^{-3}\,dx - \int 2x^{-2}\,dx$ ✓

$\frac{1}{2}v^2 = -\frac{7}{2}x^{-2} + 2x^{-1} + c$

$v = 0,\ x = \frac{1}{4}$

$0 = -\frac{7}{4}(16) + 2(4) + c$ ✓

$c = 48$

$\frac{1}{2}v^2 = -\frac{7}{x^2} + \frac{2}{x} + 48$

$v^2 = -\frac{7}{x^2} + \frac{4}{x} + 96$

$v = \pm\sqrt{\frac{-7 + 4x + 96x^2}{x^2}}$ ✓

pos

$v = \frac{1}{x}\sqrt{96x^2 + 4x - 7}$ $\quad$ 3

3) a) $N = 1000 + Ae^{-kt}$

$\frac{dN}{dt} = -kAe^{-kt}$

$= -k(N - 1000)$ $\quad$ 1

b) $t = 0$

$2500 = 1000 + A$

$\therefore A = 1500$

$N = 1000 + 1500e^{-kt}$

$2200 = 1000 + 1500e^{-2k}$

$\frac{4}{5} = e^{-2k}$ $\quad$ 2

$\ln\frac{4}{5} = -2k$

$k = -\frac{1}{2}\ln\frac{4}{5}$

c) $1300 = 1000 + 1500e^{-kt}$

$\frac{1}{5} = e^{-kt}$

$t = -\frac{1}{k}\ln\frac{1}{5}$ $\quad$ 2

$t \approx 14.4$

$t = 14\frac{1}{2}$ years