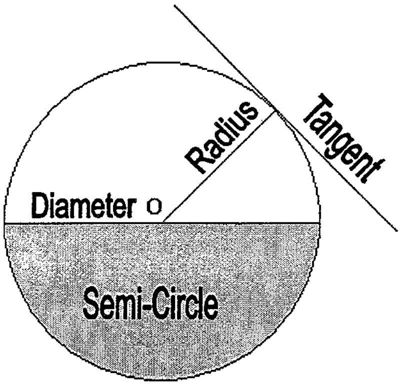
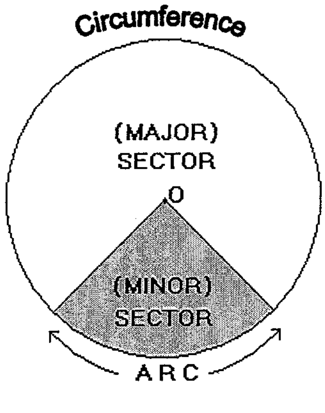
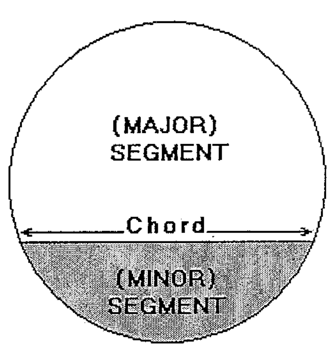
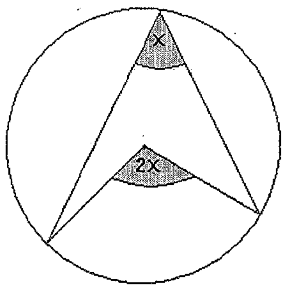
CIRCLE PROPERTIES

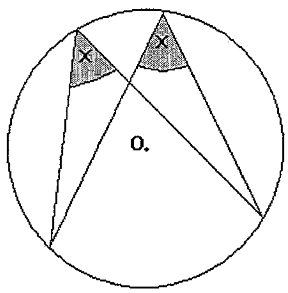




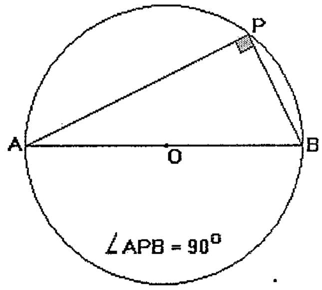




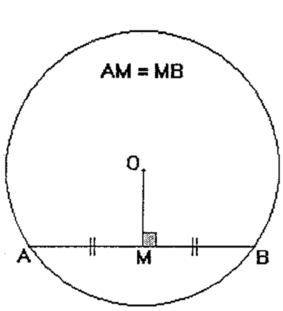
The angle at the centre is twice the angle at the circumference, when made from the same arc.



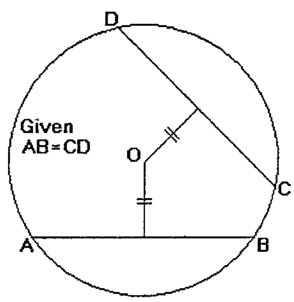
Angles at the circumference, made from the same arc, are equal.



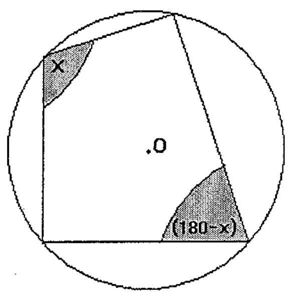
The angle in a semi-circle is a right-angle.



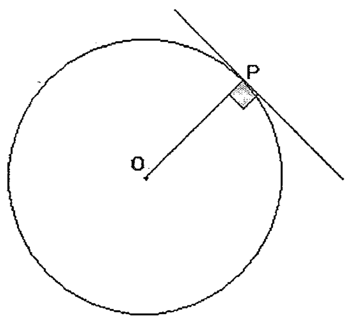
A line drawn from the centre to the mid-point of a chord, meets it at right-angles (90^{o})



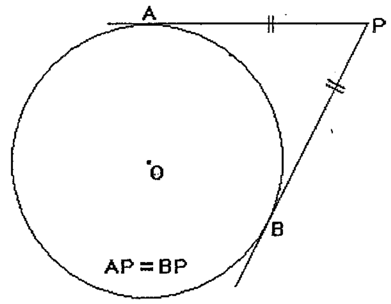
Chords of equal length are equidistant from the centre of the circle.



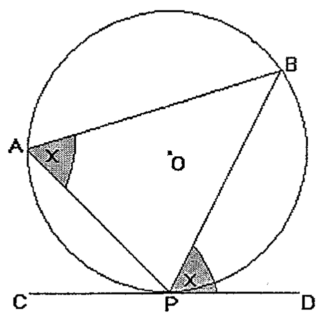
The opposite angles of a cyclic quadrilateral, are supplementary.



A Tangent meets a radius at its point of contact at right-angles.



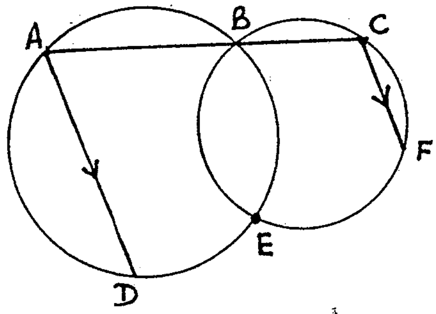
Two tangents drawn from an external point are equal in length.



The angle between a chord and a tangent is equal to the angle in the alternate segment.

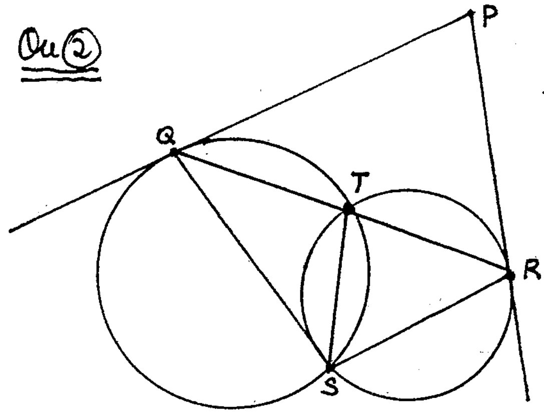
CIRCLE PROPERTIES ASSIGNMENT.

Qu ①



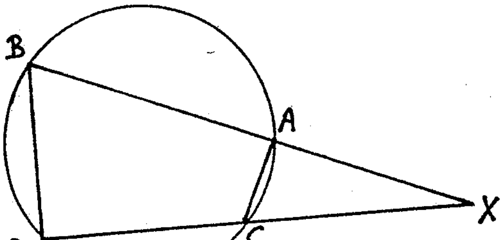
Use appropriate constructions to prove that D, E, & F are co-linear (ie $\angle DEF = 180°$)
(You are given: $AD \parallel CF$ and ABC is a straight line.)

Qu ②



PQ and PR are Tangents at Q & R.
QTR is a straight line. (ie $\angle QTR = 180°$)
Prove that PQSR is a cyclic quadrilateral.
(Hint: Use angle in alternate segment to prove its opposite angles are supplementary)

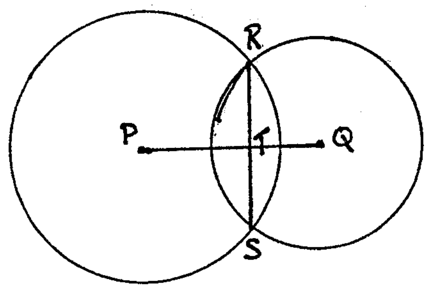
u ③



Prove ΔACX is similar to ΔBDX
Hence show that $XA.XB = XC.XD$

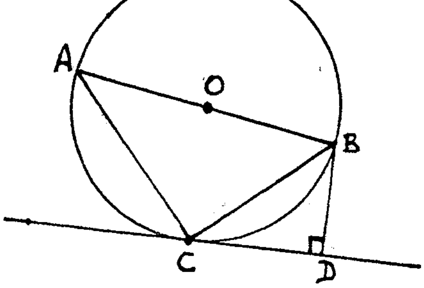
(

Qu ④



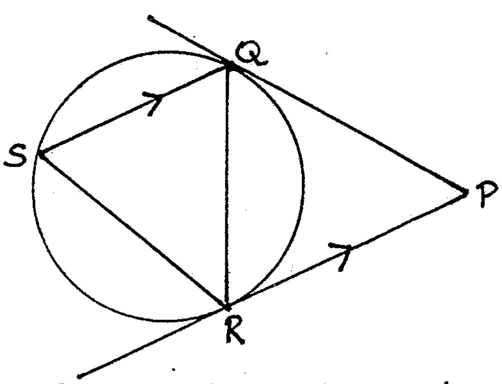
P and Q are the centres of two circles which intersect at points R and S.
Prove that $PQ \perp RS$
(Hint: Contruct 4 radii PR, PS, QR, QS and use Congruent Δ's)

⑤



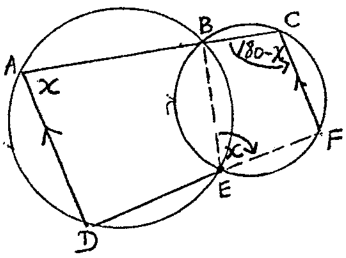
en O, is the centre of the Circle
> is a tangent.
d $CD \perp BD$
ve that: ΔACB is similar to ΔBCD
ce prove that $BC^2 = BD \cdot AB$

Qu ⑥



Given PQ and PR are tangents at Q & R.
Given $SQ \parallel RP$
PROVE: (i) ΔSRQ is isosceles
(ii) $\angle QRS = \angle QPR$

Quest ①



Construct DE, BE and FE

in To prove $\angle DEB + \angle FEB = 180°$

Let $\angle BAD = x$

Then $\angle BEF = x$ (exter. angle of a cyclic quadr. = its interior oppos)

Also since $AD \parallel CF$ (given)

Then $\angle ACF = 180 - x$ (co-interior $\angle$'s are supplementary)

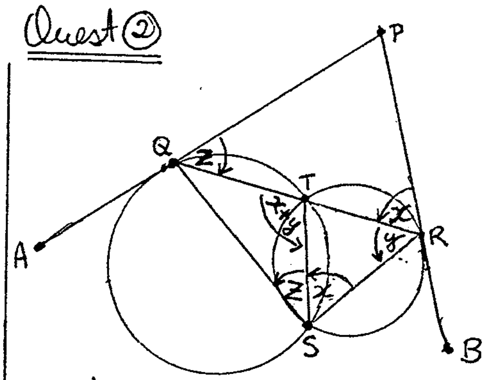
Also $\angle DEB = 180 - x$ (ext. angle of a cyclic quad. = int. opp.)

$\therefore \angle DEB + \angle FEB = (180-x)° + x° = 180°$

$\therefore$ DEF is a straight line

$\therefore$ D, E, & F are co-linear. .... proven.

Quest ②



Aim: To prove PQSR is a cyclic quadrilateral ie its opposite $\angle$'s are supplementary!

Label A & B as above.

Let $\angle PRQ = x$

Then $\angle RST = x$ (Angle in the Alternate segmate.)

Let $\angle SRT = y$

$\therefore \angle QTS = x + y$ (exterior $\angle$ of a Δ = sum of 2 inter. opps)

Let $\angle PQR = z$

Then $\angle QST = z$ (Angle in the Alternate Segment)

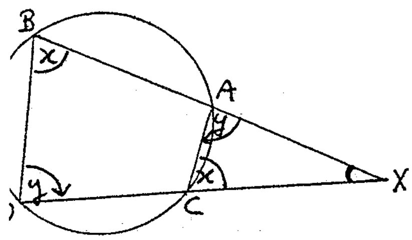
Also $\angle TQS = 180° - (x+y+z)$ (Angle Sum of $\Delta QTS = 180$)

$\therefore \angle PQS = [180 - x - y - \cancel{z}] + \cancel{z}$ (adjacent angles)

$= 180 - (x+y)$

$=$ Supplement of $\angle PRS$ .... proven.

uest ③



: To prove ΔACX similar to ΔBDX

$\angle AXC$ is common

$\angle ACX = \angle DBX = \cancel{x}$ (exter. $\angle$ of quad = int. opp.)

ilarily:

$\angle CAX = \angle BDX = y$ (" " " " " ")

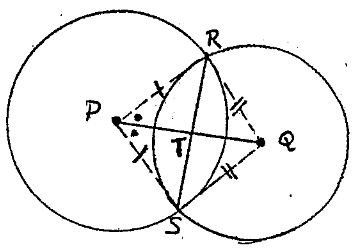
$\Delta ACX \,|||\, \Delta BDX$ (3 angles equals)

all corresponding sides in same ratio (prop.)

$\underbrace{\frac{BD}{AC} = \frac{XB}{XC} = \frac{XD}{XA}}_{\text{from this....}}$ cross-multiply ... (ie multiply by $XA \cdot XC$)

$XA \cdot XB = XC \cdot XD$ proven.

Quest ④



Aim: To first show $\Delta PRQ \equiv \Delta PSQ$ — (1)

& then show $\Delta PRT \equiv \Delta PST$ — (2)

(1) Construct PR = PS (radii)

and RQ = SQ (radii)

Also PQ is common

$\therefore \Delta PRQ \equiv \Delta PSQ$ (SSS)

$\therefore$ All corresponding sides & angles are equal !!

In particular:- $\angle RPT = \angle SPT$

(2) In ΔPTR an ΔPTS ...

PR = PS (radii)

$\angle RPT = \angle SPT$ (proven in (1))

& PT is common side.

$\therefore \Delta RPT \equiv \Delta SPT$

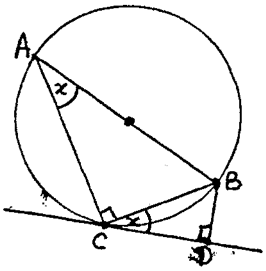
$\therefore$ All corresponding sides & angles are equal!!

In particular RT = ST

$\therefore$ PQ bisects RS

and $\angle RTP = \angle STP$ But $\angle RTP + \angle STP = 180°$

Quest ⑤



Aim: To prove ΔCBD similar to ΔACB

$\angle BDC = 90°$ (given)

$\angle ACB = 90°$ (angle in a semi-circle $=90°$)

Also: $\angle BCD = \angle CAB$ (angle in the alternate segment)

$\therefore \angle ABC = \angle CBD = 90 - x$ (angle sum of $\Delta = 180°$)

$\therefore \Delta ACB \,|||\, \Delta BCD$ (all 3 angles equal)

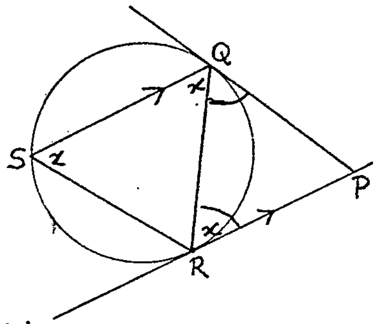
All corresponding sides are in same ratio!

$$\underbrace{\frac{BC}{AB} = \frac{BD}{BC}}_{} = \frac{CD}{AC}$$

using these & cross-multiplying....

$BC^2 = BD \cdot AB$ ---- proven.

Quest ⑥



Aim: (1) To prove ΔSRQ is isosceles

(2) To prove $\angle QRS = \angle QPR$

(1) Let $\angle PRQ = x$ Since $SQ \parallel RP$ (given)

Then $\angle SQR = x$ (alternate to $\angle QRP$)

Also $\angle QSR = x$ (angle in the alternate segment)

$\therefore \Delta SRQ$ is isosceles

$\therefore SR = QR$ (2 sides equal).

(2) $QP = RP$ (2 Tangents to a circle from an external point are equal.)

$\therefore \Delta QPR$ is isosceles.

$\therefore \angle RQP = \angle QRP = x$

But $\angle SRQ = 180 - 2x$ (angle sum $\Delta SRQ = 180°$)

& $\angle QPR = 180 - 2x$ (angle sum $\Delta QPR = 180°$)

$\therefore \angle SRQ = \angle QPR$ --- proven