

YEAR 11 EXTENSION 1 TRIGONOMETRY TEST (CRANBROOK)

June 2003

HRK

TO BE COLLECTED IN 3 SECTIONS

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PART A Marked by HRK

- 1) If $\tan J$ is negative and $\operatorname{cosec} J = \frac{5}{3}$, find the value of $\sec J$. 3

- 2) Find the value of θ if $\operatorname{cosec} 42 = \sec(78 - 3\theta)^\circ$ 2

- 3) i. Show that $\frac{1 + \cos 2A}{\sin 2A} = \cot A$. 4

- ii. Hence find the exact value of $\cot^\circ 15$. 3

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PART B Marked by DGS

- 1) Find all angles θ with $0 \leq \theta \leq 360^\circ$ for which $\sin 2\theta = \sin \theta$. 4

- 2) i. Express $\sin 4t + \sqrt{3} \cos 4t$ in the form $R \sin(4t + \alpha)$, where α is in degrees. 3

- ii. Hence, or otherwise, find the general solution in exact form of the equation $\sin 4t + \sqrt{3} \cos 4t = 0$. 3

- 3) A yacht sailing due west, turns at A to avoid a treacherous reef and sails on a course bearing $212^\circ 20'$ for 2.8 nautical miles to B . It then turns and sails on a course bearing $330^\circ 35'$ to a point C , due west of A .

Find to the nearest tenth of a nautical mile, the distance BC . 4

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PART C Marked by CJL

1)

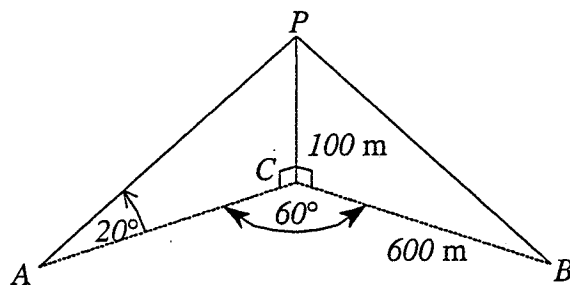


FIGURE NOT TO SCALE

Two yachts A and B subtend an angle of 60° at the base C of a cliff.
From yacht A the angle of elevation of the point P , 100 metres vertically above C , is 20° .
Yacht B is 600 metres from C .

- i. Calculate the length AC . 2
- ii. Calculate the distance between the two yachts. 3

2)

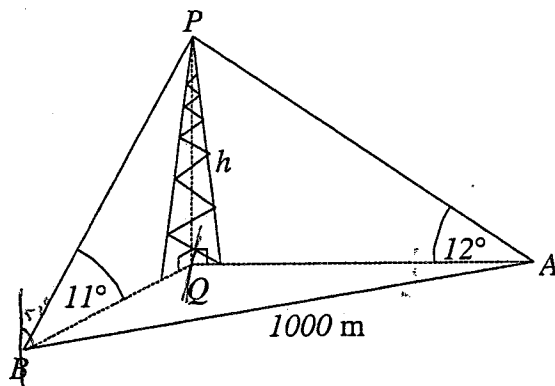


FIGURE NOT TO SCALE

The angle of elevation of a tower PQ of height h metres at a point A due east of it is 12° .
From another point B , the bearing of the tower is 051°T and the angle of elevation is 11° .
The points A and B are 1000 metres apart and on the same level as the base Q of the tower.

- i. Show that $\angle AQB = 141^\circ$. 2
- ii. Consider the triangle APQ and show that $AQ = h \tan 78^\circ$ 2
- iii. Find a similar expression for BQ . 2
- iv. Use the cosine rule in the triangle AQB to calculate h to the nearest metre. 3

END OF TEST

✓ = 1 MARK

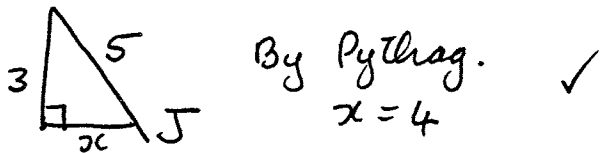
YR 11 EXT 1 TRIGONOMETRY TEST

20/40 50%
HRIK

JUNE 2003

PART A

1) $\tan J < 0$
 ③ $\operatorname{cosec} J = +$ } \therefore 2nd Q
 \therefore Sec is NEG.



$\therefore \sec J = -\frac{5}{4}$

2) $\operatorname{cosec} 42^\circ = \sec(78-3\theta)^\circ$

② CO-RATIOS
 $\therefore 42^\circ + 78^\circ - 3\theta = 90^\circ$
 $3\theta = 30^\circ$
 $\therefore \theta = 10^\circ$

3) (i) $1 + \cos 2A$

LHS = $\frac{1 + \cos 2A}{\sin 2A}$

④ = $\frac{1 + 2\cos^2 \theta - 1}{2 \sin \theta \cos \theta}$
 = $\frac{\cos \theta}{\sin \theta} = \cot \theta = \text{RHS}$

(ii) ③ $\cot 15^\circ = \frac{1 + \cos 30^\circ}{\sin 30^\circ}$

= $\frac{1 + \frac{\sqrt{3}}{2}}{\frac{1}{2}}$

= $2 + \sqrt{3}$

PART B

④ $\sin 2\theta - 2\sin \theta = 0$
 $2 \sin \theta \cos \theta - 2 \sin \theta = 0$
 $\sin \theta (2 \cos \theta - 2) = 0$

$\sin \theta = 0$ $2 \cos \theta - 2 = 0$

$\theta = 0^\circ, 180^\circ, 360^\circ$ $\cos \theta = 1$

$\theta = 60^\circ, 300^\circ$

$\therefore \theta = 0^\circ, 60^\circ, 180^\circ, 300^\circ, 360^\circ$

2/1 (i) $R = \sqrt{1^2 + \sqrt{3}^2} \tan \alpha = \sqrt{3}$
 ③ = 2 $\alpha = 60^\circ$

$\therefore R \sin(4t + \alpha) = 2 \sin(4t + 60^\circ)$

(ii) ③ $2 \sin(4t + 60^\circ) = 0$

$\sin(4t + 60^\circ) = 0$

$4t + 60^\circ = 180n$

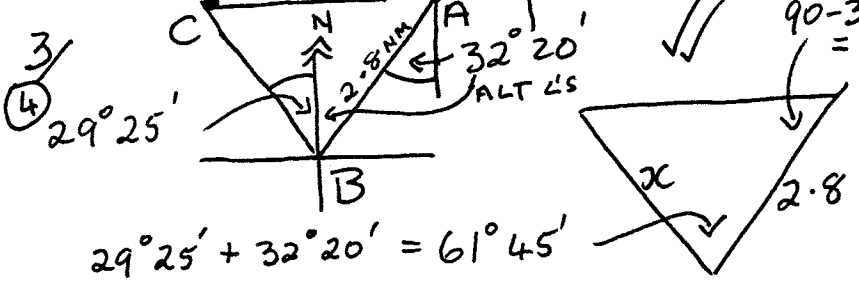
$\frac{4t}{4} = \frac{180n - 60}{4}$

$\therefore t = 45n - 15$ (n ∈ J)

OR OTHERWISE: Since RHS is zero simply use tan!!!

$\frac{\sin 4t}{\cos 4t} = -\sqrt{3}$

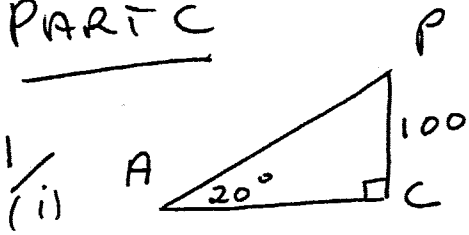
$\therefore \tan 4t = -\sqrt{3}$, $4t = 180n - 60$
 $t = 45n - 15$ AS BEFORE



$\frac{x}{\sin 57^\circ 40'} = \frac{2.8}{\sin 60^\circ 35'}$

$\therefore x = \frac{2.8 \times \sin 57^\circ 40'}{\sin 60^\circ 35'}$
 $= 2.7 \text{ nm}$

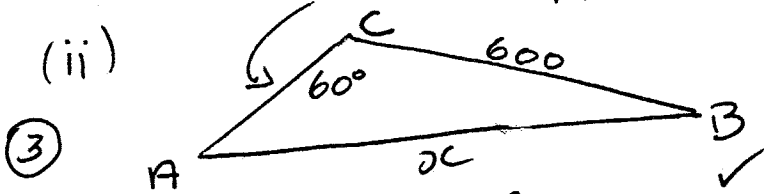
PART C



(2) $\tan 20^\circ = \frac{100}{AC}$ ✓

$\therefore C = \frac{100}{\tan 20^\circ}$ ✓

$= 274.7 \text{ m (1 d/p)}$



(3)

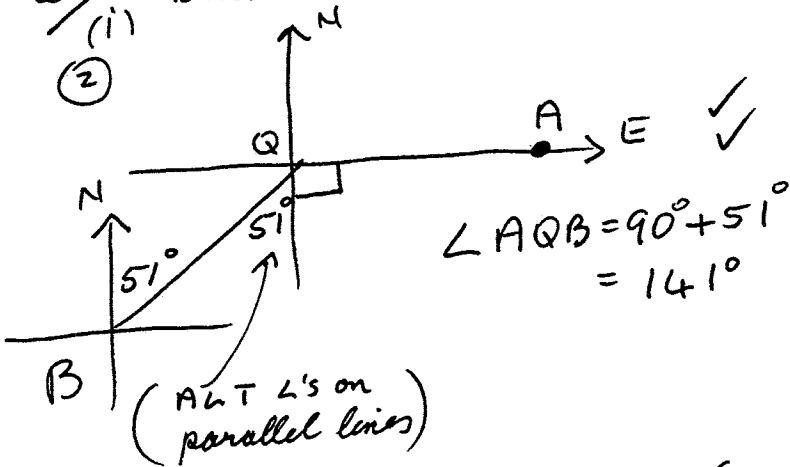
use cosine rule

$x^2 = (274.7)^2 + 600^2 - 2 \times 274.7 \times 600 \times \cos 60^\circ$

$= 270640.09$

$\therefore x = 520 \text{ m (nearest m)} \checkmark (520.2308045!)$

2/ (i) BIRD'S EYE VIEW!



(ii) In ΔAPQ , $\angle APQ = 78^\circ$ (\angle sum of Δ) ✓✓

(2) $\tan 78^\circ = \frac{AQ}{h} \therefore AQ = h \tan 78^\circ$

(iii) (2) In ΔBQP , $\tan 79^\circ = \frac{BQ}{h}$ ✓
 $\therefore BQ = h \tan 79^\circ$

(iv) $1000^2 = h^2 \tan^2 78^\circ + h^2 \tan^2 79^\circ - 2 \times h \tan 78^\circ \times h \tan 79^\circ \cos 141^\circ$ ✓

(3) $h^2 = \frac{1000^2}{\tan^2 78^\circ + \tan^2 79^\circ - 2 \tan 78^\circ \tan 79^\circ \cos 141^\circ} \therefore h = 108 \text{ m}$ ✓✓

$h^2 = 11598.39101$
 $\therefore h = 107.695826$

