YEAR 11 EXTENSION 1 TRIGONOMETRY TEST (CRANBROOK)

June 2003 HRK

TO BE COLLECTED IN 3 SECTIONS

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PART A Marked by HRK

1) If $\tan J$ is negative and $\operatorname{cosec} J = \frac{5}{3}$, find the value of $\sec J$. 3

2) Find the value of θ if $\operatorname{cosec} 42 = \sec(78 - 3\theta)^\circ$ 2

3) i. Show that $\frac{1 + \cos 2A}{\sin 2A} = \cot A$. 4

ii. Hence find the exact value of $\cot 15^\circ$. 3

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PART B Marked by DGS

1) Find all angles θ with $0 \leq \theta \leq 360^\circ$ for which $\sin 2\theta = \sin \theta$. 4

2) i. Express $\sin 4t + \sqrt{3}\cos 4t$ in the form $R\sin(4t + \alpha)$, where α is in degrees. 3

ii. Hence, or otherwise, find the general solution in exact form of the equation $\sin 4t + \sqrt{3}\cos 4t = 0$. 3

3) A yacht sailing due west, turns at A to avoid a treacherous reef and sails on a course bearing $212^\circ 20'$ for $2{\cdot}8$ nautical miles to B. It then turns and sails on a course bearing $330^\circ 35'$ to a point C, due west of A.

Find to the nearest tenth of a nautical mile, the distance BC. 4

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PART C Marked by CJL

1)

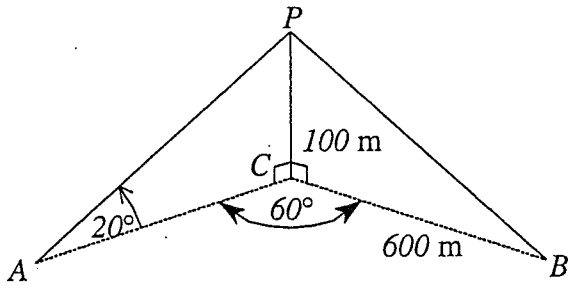


FIGURE NOT TO SCALE

Two yachts A and B subtend an angle of $60°$ at the base C of a cliff.
From yacht A the angle of elevation of the point P, 100 metres vertically above C, is $20°$.
Yacht B is 600 metres from C.

i. Calculate the length AC. 2

ii. Calculate the distance between the two yachts. 3

2)

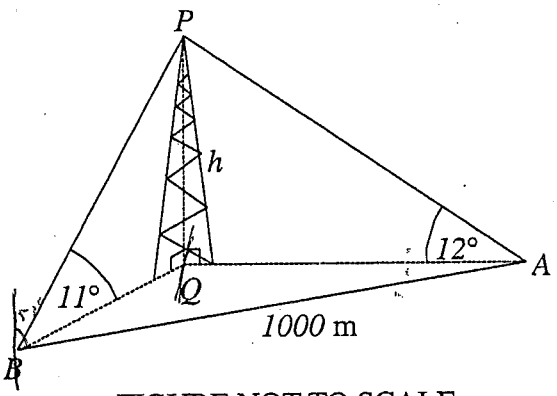


FIGURE NOT TO SCALE

The angle of elevation of a tower PQ of height h metres at a point A due east of it is $12°$.
From another point B, the bearing of the tower is $051°$T and the angle of elevation is $11°$.
The points A and B are 1000 metres apart and on the same level as the base Q of the tower.

i. Show that $\angle AQB = 141°$. 2

ii. Consider the triangle APQ and show that $AQ = h \tan 78°$ 2

iii. Find a similar expression for BQ. 2

iv. Use the cosine rule in the triangle AQB to calculate h to the nearest metre. 3

END OF TEST

√=1 MARK

YR 11 EXT 1 TRIGONOMETRY TEST

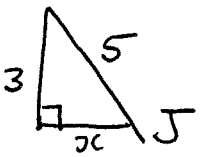
JUNE 2003

21/40 53%

1+RK

PART A

1) ③ $\tan J < 0$ } ∴ 2nd Q
$\operatorname{cosec} J = +$ } ∴ Sec is NEG. ✓



By Pythag. ✓
$x = 4$

$\therefore \sec J = -\frac{5}{4}$ ✓

2) ② $\operatorname{cosec} 42^\circ = \sec(78-3\theta)^\circ$

CO-RATIOS ✓

$\therefore 42^\circ + 78^\circ - 3\theta = 90^\circ$

$3\theta = 30^\circ$

$\therefore \theta = 10^\circ$ ✓

3) (i) ④ $LHS = \dfrac{1+\cos 2A}{\sin 2A}$

$= \dfrac{1+2\cos^2\theta - 1}{2\sin\theta\cos\theta}$ ✓ ✓ ✓✓

$= \dfrac{\cos\theta}{\sin\theta} = \cot\theta = RHS$ //

(ii) ③ $\cot 15^\circ = \dfrac{1+\cos 30^\circ}{\sin 30^\circ}$ ✓

$= \dfrac{1+\frac{\sqrt{3}}{2}}{\frac{1}{2}}$ ✓

$= 2+\sqrt{3}$ ✓

PART B

1/ ④ $\sin 2\theta - 2\sin\theta = 0$

$2\sin\theta\cos\theta - 2\sin\theta = 0$

$2\sin\theta(\cos\theta - 1) = 0$

$\sin\theta = 0$: $\theta = 0^\circ, 180^\circ, 360^\circ$

$2\cos\theta - 1 = 0$: $\cos\theta = \frac{1}{2}$, $\theta = 60^\circ, 300^\circ$

$\therefore \theta = 0^\circ, 60^\circ, 180^\circ, 300^\circ, 360^\circ$

2/ (i) ③ $R = \sqrt{1^2+\sqrt{3}^2} = 2$ ✓ $\tan\alpha = \sqrt{3}$, $\alpha = 60^\circ$ ✓

$\therefore R\sin(4t+\alpha) = 2\sin(4t+60^\circ)$ ✓

(ii) ③ $2\sin(4t+60^\circ) = 0$

$\sin(4t+60^\circ) = 0$ ✓

$4t + 60^\circ = 180^\circ n$ ✓

$\frac{4t}{4} = \frac{180n-60}{4}$ ✓

$\therefore t = 45n - 15$ $(n \in J$

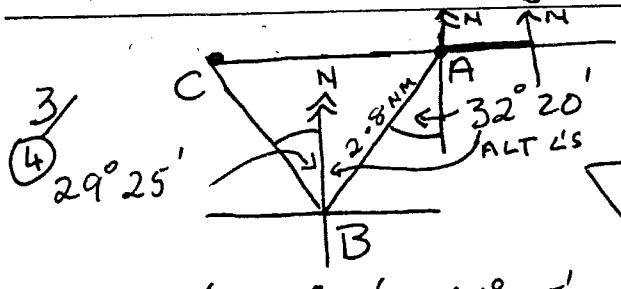
OR OTHERWISE: Since RHS is zero SIMPLY use tan !!! ☺

ie $\dfrac{\sin 4t}{\cos 4t} = -\sqrt{3}$

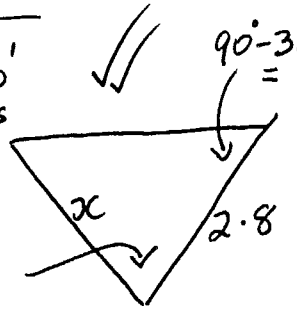
$\therefore \tan 4t = -\sqrt{3}$, $4t = 180n - 60$

$t = 45n - 15$ AS BEFORE //

3/ ④



$29^\circ 25' + 32^\circ 20' = 61^\circ 45'$



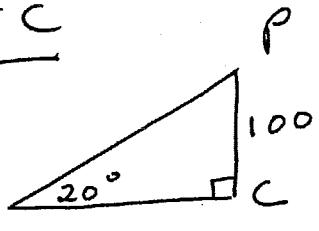
$\dfrac{x}{\sin 57^\circ 40'} = \dfrac{2\cdot 8}{\sin 60^\circ 35'}$ ✓

$\therefore x = \dfrac{2\cdot 8 \times \sin 57^\circ 40'}{\sin 60^\circ 35'}$

$= 2\cdot 7$ nm // ✓

PART C

1/ (i)



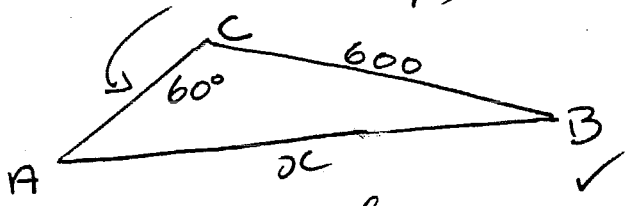
(2) $\tan 20° = \frac{100}{AC}$ ✓

$\therefore AC = \frac{100}{\tan 20°}$ ✓

$= 274.7\text{m}$ (1 d/p)

(ii)

(3)



✓

use cosine rule

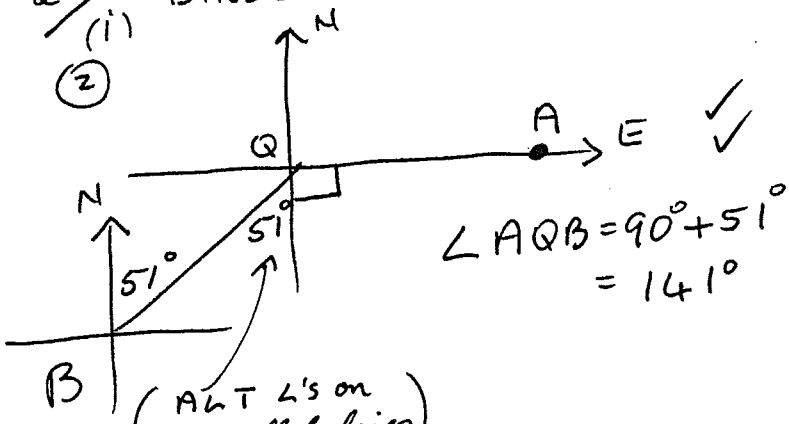
$x^2 = (274.7)^2 + 600^2 - 2 \times 274.7 \times 600 \times \cos 60°$ ✓

$= 270640.09$

$\therefore xc = 520\text{m}$ (nearest m) ✓ (520.23080451)

2/ (i) BIRD'S EYE VIEW!

(2)



✓✓

$\angle AQB = 90° + 51°$

$= 141°$

(ii) In $\triangle APQ$, $\angle APQ = 78°$ ($\angle$ sum of $\triangle$) ✓✓

(2) $\tan 78° = \frac{AQ}{h}$ $\therefore AQ = h \tan 78°$

$h^2 = 11598.39101$

$\therefore h = 107.695826$

(iii) (2) In $\triangle BQP$, $\tan 79° = \frac{BQ}{h}$ ✓✓

$\therefore BQ = h \tan 79°$

(iv) (3) $1000^2 = h^2 \tan^2 78° + h^2 \tan^2 79 - 2 \times h \tan 78° h \tan 79° \cos 141°$ ✓

$h^2 = \frac{1000^2}{\tan^2 78° + \tan^2 79° - 2 \tan 78° \tan 79° \cos 141°}$ ✓✓

$\therefore h = 108\text{m}$