WITH SOL'NS

CRANBROOK
SCHOOL

Year 12 Extension 1 Mathematics

Mini Examination

Friday September 5, 2008

Instructions

- There are three questions, each worth 12 marks
- Attempt all questions
- Answer each question in a new booklet
- Show all necessary working
- Calculators are allowed in all sections

Time Allowed: 45 minutes   Total Marks: 36

Typeset by JSH

Checked by HRK, SKB, CJC

Question 1 (12 Marks) **START A NEW BOOKLET** **Marked by HRK**

(a) An F18 jet is climbing at a speed of 140 m/s at an angle of 30° to the horizontal. When the jet is 600 metres above the ocean, it drops a flare from the wing. The only force acting on the flare is gravity.

(i) Find the time taken for the flare to hit the ocean. **3**

(ii) Calculate the maximum height reached by the flare. **2**

(iii) What is the horizontal distance travelled by the flare? **1**

(b) A particle is projected horizontally with velocity, V ms^{-1}, from a point h metres above the ground. Take g ms^{-2} as the acceleration due to gravity.

(i) Taking the origin at the point on the ground immediately below the projection point, find expressions for x and y, the horizontal and vertical displacements respectively of the particle at time t seconds. **2**

(ii) Hence show that the equation of the path of the particle is given by the equation, $y = \dfrac{2hV^2 - gx^2}{2V^2}$. **2**

(iii) Find how far the particle travels horizontally from its point of projection before it hits the ground. **2**

Question 2 (12 Marks) **START A NEW BOOKLET** **Marked by CJC**

(a) In the expansion of $\left(\dfrac{3}{x} - 5x^2\right)^9$ find the term independent of x. **3**

(b) Angelos loves to play basketball. From the free throw line he makes 3 out of every 5 shots. For every basket he makes he scores one point.

(i) In a game against Trinity he had 8 free throws. What is the probability that he scored 2 points? **1**

(ii) How many free throws would he need in one game so that the probability that he scores at least one point is 0.9978? **2**

(c) (i) Write down the binomial expansion of $(1+x)^n$ in ascending powers of x. **1**

(ii) Show that $\sum_{r=1}^{n} {}^nC_r = 2^n - 1$ **1**

(iii) By using integration and the answer in part (i), show that

$$\frac{1}{n+1}\sum_{r=1}^{n+1} {}^{n+1}C_r = \sum_{r=0}^{n} \frac{{}^nC_r}{r+1}$$ **4**

Question 3 (12 Marks) **START A NEW BOOKLET** **Marked by JSH**

(a) How many different 4-digit numbers may be formed from 1, 2, 3, 4, 5, 6 if;

- **(i)** none of the digits are repeated? **1**
- **(ii)** the digits may be repeated **1**
- **(iii)** the last digit is a multiple of 3? **1**
- **(iv)** the number is even **1**

(b) There are 12 videotapes arranged in a row on a shelf in a video shop. There are 3 identical copies of *Gone with the Wind,* 4 of *Tootsie* and 5 of *Gladiator.*

- **(i)** How many different arrangements of the videotapes are there? **1**
- **(ii)** How many different arrangements are there if the videos with the same title are grouped together? **1**
- **(iii)** The 12 videotapes are arranged at random in a row on the shelf. Find the probability that the arrangement has a copy of *Gone with the Wind*, at one end of the row, and a copy of *Gladiator* at the other end. **2**

(c) Ten people arrive to eat at a restaurant. The only seating available for them is at two circular tables, one that seats six persons, and another that seats four. Using these tables, how many different seating arrangements are there for the ten people? **2**

(d) In how many ways can 7 people sit at a round table so that 2 particular people:

- **(i)** sit next to each other? **1**
- **(ii)** are separated? **1**

END OF EXAMINATION ☺

1c

$\ddot{x} = 0$ $\qquad$ $\ddot{y} = -10$

$\dot{x} = V\cos\alpha$ $\qquad$ $\dot{y} = -10t + V\sin\alpha$

$x = Vt\cos\alpha$; $y = -5t^2 + Vt\sin\alpha + 600$

Also 504 km/hr $= 140$ m/s

$\therefore \dot{x} = 70\sqrt{3}$ $\qquad$ $\dot{y} = -10t + 70$

$x = 70\sqrt{3}.t$ $\qquad$ $y = -5t^2 + 70t + 600$

) $y = 0 \Rightarrow -5t^2 + 70t + 600 = 0$

$-5(t^2 - 14t - 120) = 0$

$-5(t-20)(t+6) = 0$

$\Rightarrow t = 20$ seconds

(ii) $\dot{y} = 0 \Rightarrow -10t + 70 = 0$

$t = 7$

At $t=7$, $y = -5\times7^2 + 70\times7 + 600$

$= 845$ metres

iii) At $t=20$, $x = 70\sqrt{3}\times20$

$= 2424.87$

$\doteqdot 2.425$ kilometres

1b

i. In the x direction:

$$\ddot{x} = 0 \Rightarrow \dot{x} = \int 0\,dt = C_1$$

When $t = 0$, $\dot{x} = V \Rightarrow C_1 = V$

$$\therefore \dot{x} = V$$

$$x = \int V\,dt = Vt + C_2$$

When $t = 0$, $x = 0 \Rightarrow C_2 = 0$

$$\therefore x = Vt$$

In the y direction:

$$\ddot{y} = -g \Rightarrow \dot{y} = \int -g\,dt = -gt + C_3$$

When $t = 0$, $\dot{y} = 0 \Rightarrow C_3 = 0$

$$\therefore \dot{y} = -gt$$

$$y = \int -gt\,dt = -\tfrac{1}{2}gt^2 + C_4$$

When $t = 0$, $y = h \Rightarrow C_4 = h$

$$\therefore y = -\tfrac{1}{2}gt^2 + h$$

ii. $x = Vt \Rightarrow t = \dfrac{x}{V}$. Substitute into $y = -\tfrac{1}{2}gt^2 + h$

$$y = -\tfrac{1}{2}g \times \left(\frac{x}{V}\right)^2 + h$$

$$= \frac{-gx^2}{2V^2} + h$$

$$= \frac{-gx^2 + 2V^2h}{2V^2}$$

iii. We require $y = 0$ thus $\dfrac{-gx^2 + 2V^2h}{2V^2} = 0 \Rightarrow x^2 = \dfrac{2V^2h}{g} \Rightarrow x = \pm\sqrt{\dfrac{2V^2h}{g}}$

But the particle is moving in a positive direction so $x = V\sqrt{\dfrac{2h}{g}}$

$) \; T_{k+1} = {}^nC_k a^{n-k} b^k$

${}^9C_k \left(\frac{3}{x}\right)^{9-k} (-5x^2)^k$ ✓

${}^9C_k \; 3^{9-k} x^{k-9} (-5)^k x^{2k}$

${}^9C_k \; 3^{9-k} x^{3k-9} (-5)^k$

i.e. $3k - 9 = 0$

$3k = 9$

$k = 3$ ✓

$\therefore$ term is $T_4 = {}^9C_3 \; 3 \; (-5)$

$= 84 \times 729 \times -125$

$= -7\,654\,500$ ✓

(b) $P(S) = \frac{3}{5} = p$

(i) $P(\bar{S}) = \frac{2}{5} = q$

$(p+q)^8$

$P(2 \text{ points}) = {}^8C_6 \, p^2 q^6$

$= {}^8C_6 \left(\frac{3}{5}\right)^2 \left(\frac{2}{5}\right)^6$ ✓

$= 28 \times \frac{9}{25} \times \frac{64}{15625}$

$= \frac{16\,128}{390\,625}$

$\doteqdot 0.04$ (to 2 d.p.)

(ii) $P(\text{at least one point}) = 1 - P(\text{no points})$

$\therefore 1 - \left(\frac{2}{5}\right)^n = 0.9978$ ✓

$\left(\frac{2}{5}\right)^n = 0.0022$

$n = \frac{\ln 0.0022}{\ln \frac{2}{5}}$

$n = 6.678\ldots$

$\therefore$ 7 shots are needed. ✓

Markers Notes

(a) Done very well.
Remember to use $(-5)^3$. Some students ignored the minus sign

(b) (i) Done well.
Better to give answer in exact form as a fraction, instead of a rounded decimal

(ii) This is actually a 2U question. Don't think that every question is binomial probability.

c) (i) Done well

(ii) Don't try to learn proofs off by heart. Let $x = 1$ and use part (i)

(iii) When integrating you must include "$+c$" and then evaluate it.
Generally these types of questions involve letting $x = 0$ or $x = 1$ or $x = -1$
Always a good idea to write out Σ notation so you know what you're trying to prove.

i) $(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \dots + \binom{n}{n}x^n$ — ①

i) R.T.P.

$\binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \dots \binom{n}{n} = 2^n - 1$

Let $x = 1$ in ①

$(1+1)^n = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \dots + \binom{n}{n}$

Now $\binom{n}{0} = 1$

$\therefore 2^n - 1 = \binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \dots + \binom{n}{n}$ as required.

ii) R.T.P

$$\frac{1}{n+1}\left[\binom{n+1}{1} + \binom{n+1}{2} + \binom{n+1}{3} + \dots + \binom{n+1}{n+1}\right] = \frac{\binom{n}{0}}{1} + \frac{\binom{n}{1}}{2} + \frac{1}{3}\binom{n}{2} + \dots + \frac{1}{n+1}\binom{n}{n}$$

Integrate b.s. of ①

$$\frac{(1+x)^{n+1}}{n+1} + c = \binom{n}{0}x + \frac{1}{2}\binom{n}{1}x^2 + \frac{1}{3}\binom{n}{2}x^3 + \dots + \frac{1}{n+1}\binom{n}{n}x^{n+1}$$

let $x = 0$ to find c

$$\frac{1}{n+1} + c = 0$$

$$c = -\frac{1}{n+1}$$

$$\therefore \frac{1}{n+1}(1+x)^{n+1} - \frac{1}{n+1} = \binom{n}{0}x + \frac{1}{2}\binom{n}{1}x^2 + \frac{1}{3}\binom{n}{2}x^3 + \dots + \frac{1}{n+1}\binom{n}{n}x^{n+1}$$

$$\frac{1}{n+1}\left[(1+x)^{n+1} - 1\right] = \binom{n}{0}x + \frac{1}{2}\binom{n}{1}x^2 + \frac{1}{3}\binom{n}{2}x^3 + \dots + \frac{1}{n+1}\binom{n}{n}x^{n+1}$$

let $x = 1$

$$\frac{1}{n+1}\left[2^{n+1} - 1\right] = \binom{n}{0} + \frac{1}{2}\binom{n}{1} + \frac{1}{3}\binom{n}{2} + \dots + \frac{1}{n+1}\binom{n}{n}$$

Now LHS $= \frac{1}{n+1}\left[2^{n+1} - 1\right]$

$= \frac{1}{n+1}\left[\binom{n+1}{1} + \binom{n+1}{2} + \binom{n+1}{3} + \dots + \binom{n+1}{n+1}\right]$ from (ii) using $n \to n+1$

$$\therefore \frac{1}{n+1}\sum_{r=1}^{n+1} {}^{n+1}C_r = \sum_{r=0}^{n} \frac{{}^nC_r}{n+1}$$ as required.

3 (a) (i) ${}^6P_4 = 6.5.4.3 =$ [redacted] ✓

(ii) $6^4 =$ [redacted] ✓

(iii) Assuming repetition *is* allowed

$2 \times 6^3 =$ [redacted] ✓

OR

If repetition were *not* allowed

$2 \times 5 \times 4 \times 3 =$ [redacted] ✓

(IV) $3 \times 6^3 =$ [redacted] ✓ repetition allowed

OR

$3 \times 5 \times 4 \times 3 =$ [redacted] ✓ repetition *not* allowed.

(b) (i) $\frac{12!}{3!\,4!\,5!} =$ [redacted] ✓

(ii) $3! =$ [redacted] ✓

(iii) Gone $\underbrace{\_\ \_ \ldots \_}_{10\ \text{left}}$ Gladiator

$2 \times \frac{10!}{2!\,4!\,4!} =$ [redacted] ✓

$\therefore$ Probability $= \frac{6300}{27720} =$ [redacted] ✓

(c) ${}^{10}C_6 \times 5! \times 3! =$ [redacted] ✓ ✓ 2

(d) i) fix.

$2 \times 5! =$ [redacted] ✓

(ii) $6! - 2 \times 5! =$ [redacted] ✓

Comments on Q3

3a i) ii) both done well

iii) iv) Marked correct for either interpretation with / without repetions (due to slight ambiguity of question)

b) i) mostly done well. ii) everyone got this

iii) the alternative solution $3/12 \times 5/11 + 5/12 \times 3/11 = 5/22$.

c) One mark was awarded for $5! \times 3!$ but many forgot to multiply this by ${}^{10}C_6$.

d) i) ii) } everybody got these

JSH