Name:__________________

Teacher:__________________

DANEBANK

An Anglican School for Girls

Year 12
Mathematics Extension 1
Task # 2
June, 2006

Time Allowed – 50 minutes

(This examination paper does not necessarily reflect the content or format of the final Higher School Certificate Examination Paper for this subject)

Weight: 30%

Outcomes examined: PE3, PE5, PE6, H5, H8, H9, HE4, HE6, HE7

DIRECTIONS TO CANDIDATE:

- Attempt all questions.
- Start each **new** question in on a **new page**, using the supplied paper.
- Show all necessary working otherwise full marks may not be awarded
- Marks may be deducted for careless or badly arranged work.
- All questions are NOT of equal value.
- Board approved calculators may be used.
- Write your name on this paper.
- A table of standard integrals is provided on the last page.

Question 1 (15 Marks) **Marks**

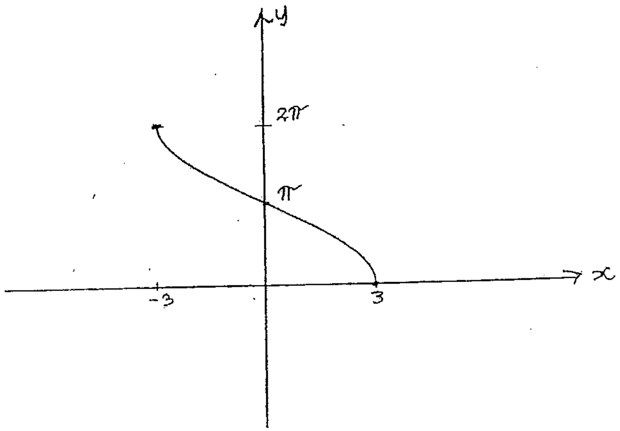
(a) Find the exact value of $\cos[\sin^{-1}(\frac{12}{13})]$ 2

(b) Differentiate: $\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)$ 3

(c) Find: $\int \frac{dx}{\sqrt{9-x^2}}$ 2

(d) Evaluate: $\int_0^4 \frac{dx}{16+x^2}$ 3

(e) Write down the equation for the following inverse trig function: 2



(f) Find the value of "a" if 3

$$\int_{-a}^{a} \sec^2 2x \; dx = 1$$

Question 2 (11 Marks) **Marks**

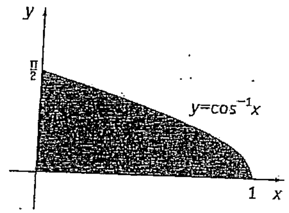
(a) $\int \frac{e^x}{1+e^{2x}}\,dx$ is equal to 1

A. $\log_e(1+e^{2x})+c$ B. $\frac{1}{2}\log_e(1+e^{2x})+c$

C. $\tan^{-1}e^{2x}+c$ D. $\tan^{-1}e^{x}+c$

(b) Draw the inverse function of the function shown on the SEPARATE ANSWER SHEET on the same graph. 2

(c)



Use Simpson's Rule with 3 function values to find an approximation to the shaded area. 3

(d) Find the general solution to $\sin 2x = \frac{1}{2}$. 2

(e) Given the equation $x - \cos x = 0$;

(i) explain how you would find your first approximation to the root of this equation. 2

(ii) If you take $x_1 = \frac{1}{2}$ as your first approximation, use one application of Newton's Method, to find a better approximation, correct to 3 decimal places. 1

Question 3 (14 Marks)

(a) Show that the equation 3

$$x^3 + x - 3 = 0$$

has a root near 1.2 and, by 'halving the interval' twice, find a better approximation to this root correct to 3 decimal places.

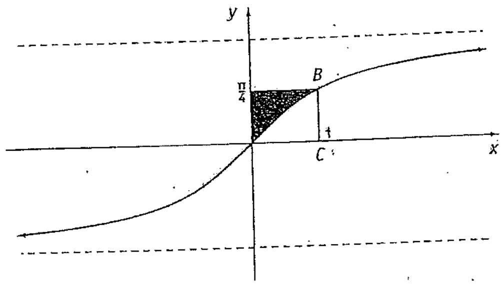
(b) Find the remainder when $P(x)=x^3 + 4x^2 - 3$ is divided by $x+1$. 1

(c) If α, β *and* γ are the roots of the equation $2x^3 - x^2 - 5x + 4 = 0$, find

(i) $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$ 2

(ii) $\alpha^2 + \beta^2 + \gamma^2$ 2

(d)



The diagram shows a sketch of the function $y=\tan^{-1} x$.

(i) Write down the domain and range of the function. 1

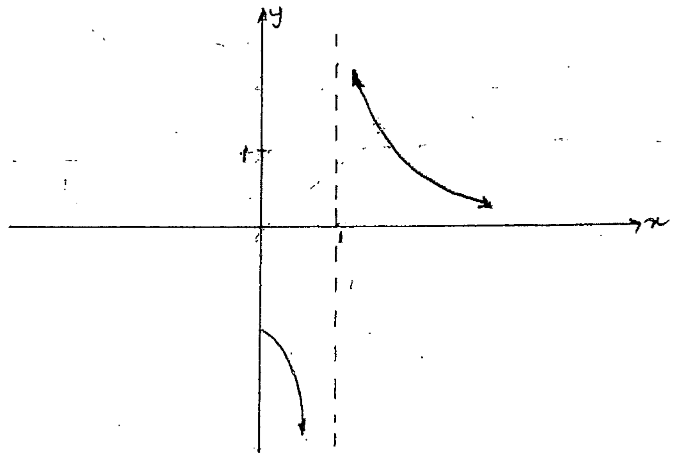
(ii) What are the coordinates of the point B? 1

(iii) Show that the exact size of the shaded on the diagram is $\frac{1}{2}\log_e 2$. 3

(iv) Hence, or otherwise, find the area bounded by $y=\tan^{-1} x$, the interval BC and the $x-axis$. 1

End of Paper.

ANSWER SHEET FOR Question 2(b) (2 Marks)

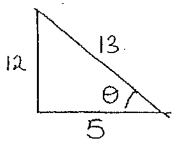


Draw the inverse function of the function shown on the same graph.

(Attach this sheet to your written answers.)

Q1

a) $\cos\left[\sin^{-1}\frac{12}{13}\right]$



$\sin^{-1}\frac{12}{13} = \theta$

$\therefore \cos\theta = \frac{5}{13}$

b) $\frac{d}{dx}\left\{\cos^{-1}\frac{1}{\sqrt{x}}\right\} = \frac{-1}{\sqrt{1-\left(\frac{1}{\sqrt{x}}\right)^2}} \times -\frac{1}{2}x^{-3/2}$

$= \frac{1}{2\sqrt{x^3}\sqrt{1-\frac{1}{x}}}$

$= \frac{1}{2\sqrt{x^3-x^2}}$

$= \frac{1}{2x\sqrt{x-1}}$

c) $\int \frac{dx}{\sqrt{9-x^2}} = \sin^{-1}\frac{x}{3} + c$

d) $\int_0^4 \frac{dx}{16+x^2} = \left[\frac{1}{4}\tan^{-1}\frac{x}{4}\right]_0^4$

$= \frac{1}{4}\tan^{-1}1 - \frac{1}{4}\tan^{-1}0$

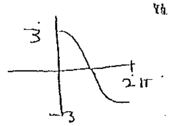
$= \frac{1}{4} \times \frac{\pi}{4}$

$= \frac{\pi}{16}$

e) range -3 to 3

domain $0 - 2\pi$

Inverse function has.



$\therefore y = 3\cos\frac{x}{2}$

$\therefore$ inverse

$x = 3\cos\frac{y}{2}$

$\frac{x}{3} = \cos\frac{y}{2}$

$y = 2\cos^{-1}\frac{x}{3}$

f) $\int_{-a}^{a} \sec^2 2x\,dx = \left[\frac{1}{2}\tan 2x\right]_{-a}^{a}$

$= \frac{1}{2}\tan 2a - \frac{1}{2}\tan(-2a)$

$= \frac{1}{2}\tan 2a + \frac{1}{2}\tan 2a$

$= \tan 2a$

$\therefore \tan 2a = 1$

$2a = \frac{\pi}{4}$

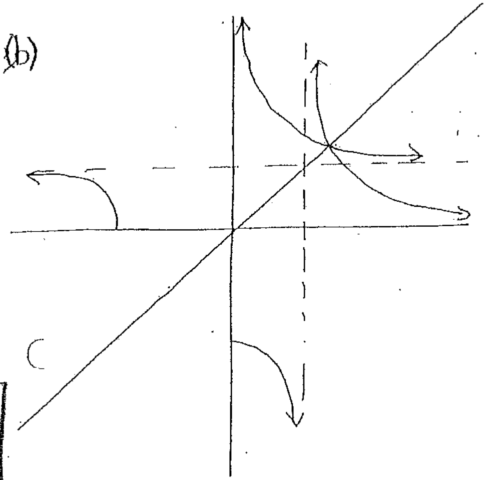
$a = \frac{\pi}{8}$

Q2

(a) $\int \frac{e^x}{1+e^{2x}}\,dx = \tan^{-1}e^x + C$

D.

(b)



(c) $h = \frac{1-0}{2}$

$= \frac{1}{2}$

x	$f(x)$		
0	$\cos^{-1}0$		$\pi/2$
$1/2$	$\cos^{-1}\frac{1}{2}$	×4	$\pi/3 \times 4$
1	$\cos^{-1}1$		0

Total $= \frac{11\pi}{6}$

$\therefore$ Area $\doteqdot \frac{1/2}{3} \times \frac{11\pi}{6}$

$= \frac{11\pi}{36}$ units^2

d) $\sin 2x = \frac{1}{2}$

$2x = \pi n + (-1)^n \sin^{-1}\frac{1}{2}$

$= \pi n + (-1)^n \times \frac{\pi}{6}$

$x = \frac{\pi n}{2} + \frac{(-1)^n \pi}{12}$

e) $x - \cos x = 0$

i) let $f(x) = x - \cos x$, try substituting different values of x until $f(x)$ is +ve for one value and -ve for anoth Take x_1 to be the midpoir of these values.

ii) $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$f(\frac{1}{2}) = \frac{1}{2} - \cos\frac{1}{2}$

$f'(x) = 1 + \sin x$

$f'(\frac{1}{2}) = 1 + \sin\frac{1}{2}$

$\therefore x_2 = \frac{1}{2} - \frac{1/2 - \cos\frac{1}{2}}{1+\sin\frac{1}{2}}$

$= 0.75522$

≈ 0.755

Q3

a) $x^3 + x - 3 = 0$

$f(1) = 1^3 + 1 - 3$
$= -1$

$f(1.5) = 1.5^3 + 1.5 - 3$
$= 1.875$

$\therefore$ there is a root between 1 & 1.5 ie near 1.2.

$f\left(\frac{1+1.5}{2}\right) = f(1.25)$
$= 1.25^3 + 1.25 - 3$
$= 0.203$

$\therefore$ the root lies between 1 & 1.25

$f\left(\frac{1+1.25}{2}\right) = f(1.125)$
$= 1.125^3 + 1.125 - 3$
$= -0.45$

$\therefore$ the root lies between 1.125 and 1.25.

ie $x = \frac{1.125 + 1.25}{2}$
$= 1.1875$
$= 1.188$

(b) $P(-1) = (-1)^3 + 4\times(-1)^2 - 3$
$= -1 + 4 - 3$
$= 0$

c) $2x^3 - x^2 - 5x + 4 = 0$

$\alpha + \beta + \gamma = \frac{1}{2}$

$\alpha\beta + \alpha\gamma + \gamma\beta = -\frac{5}{2}$

$\alpha\beta\gamma = -\frac{4}{2}$
$= -2$

i) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}$

$= \frac{-5/2}{-2}$

$= \frac{5}{4}$

ii) $\alpha^2 + \beta^2 + \gamma^2 =$

Now $(\alpha+\beta+\gamma)^2 = \alpha^2 + \beta^2 + \gamma^2 + 2\alpha\beta + 2\alpha\gamma + 2\beta\gamma$

$\therefore \alpha^2 + \beta^2 + \gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$

$= \left(\frac{1}{2}\right)^2 - 2 \times -\frac{5}{2}$

$= \frac{1}{4} + 5$

$= 5\frac{1}{4}$

d) $y = \tan x$

i) domain: all real x
range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$

ii) $\frac{\pi}{4} = \tan^{-1} x$
$x = \tan\frac{\pi}{4}$
$= 1$

$\therefore B(1, \pi/4)$

iii) Area $= \int_0^{\pi/4} f(y)\,dy$

$y = \tan^{-1} x$
$\therefore x = \tan y$

$= \int_0^{\pi/4} \tan y \, dy$

$= \int_0^{\pi/4} \frac{\sin y}{\cos y} dy$

$= \left[-\ln(\cos y) \right]_0^{\pi/4}$

$= -\ln\left(\cos\frac{\pi}{4}\right) - -\ln(\cos 0)$

$= -\ln\frac{1}{\sqrt{2}} + \ln 1$

$= -\ln 2^{-1/2} + 0$

$= \frac{1}{2}\ln 2.$

iv) Area $= 1 \times \frac{\pi}{4} - \frac{1}{2}\ln 2$

$= \frac{\pi}{4} - \frac{1}{2}\ln 2$ units^2