

4'04

EXTENSION 1 TEST 28-5-03

Trigonometric Functions, Inverse Functions and Inverse Trigonometric Functions.

Name _____ Class _____

Instructions: Show all necessary working throughout the test on A4 paper.

Begin a new page as specified.

Time allowed: 45 minutes

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SKB

1. (a) Simplify to a single termed expression: $\frac{2 \sin(\frac{\pi}{2} - 2\theta) - 1}{\sec \theta}$ [4]
- (b) Find the general solution of: $\sqrt{3} \sin x - \cos x = \sqrt{3}$ [4]
- (c) Find $\int \cos^2 x + \frac{1}{\cos^2 x} dx$ [3]
- (d) Find the exact volume generated when the area bounded by the curve $y = \sin 2x$, the x -axis and lines $x = 0$ and $x = \frac{\pi}{6}$, is rotated about the x -axis. [4]

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HRK

2. (a) A function is given by the rule $f(x) = \frac{x+1}{x+2}$. Find the rule for the inverse function $f^{-1}(x)$. [2]
- (b) Find the value of the expression $\cos^{-1}(-\frac{1}{2}) - \sin^{-1}(-\frac{1}{2})$ in terms of π . [2]
- (c) (i) Sketch the graph of $y = 4 \sin^{-1}(\frac{x}{2})$. [2]
- (ii) Find the exact equation of the tangent to the curve $y = 4 \sin^{-1}(\frac{x}{2})$ at the point where $x = 1$. [3]
- (d) Find $\frac{d}{dx} [e^{\tan^{-1}(\ln(\cos 2x))}]$ in its simplest form. [3]
- (e) Show that $\int_1^{\sqrt{2}} \frac{dx}{\sqrt{4-2x^2}} = 6 \int_{\sqrt{2}}^{\sqrt{6}} \frac{dx}{4+2x^2}$. [3]

$$a) \frac{2 \sin\left(\frac{\pi}{2} - 2\theta\right) - 1}{\sec \theta}$$

$$= \frac{2 \cos 2\theta - 1}{\sec \theta}$$

$$= \frac{2(2\cos^2 \theta - 1) - 1}{\sec \theta}$$

$$= \frac{4\cos^2 \theta - 2 - 1}{\sec \theta}$$

$$= (4\cos^2 \theta - 3) \cos \theta$$

$$= 4\cos^3 \theta - 3\cos \theta$$

$$= \cos 3\theta$$

$$b) \sqrt{3} \sin x - \cos x = \sqrt{3}$$

$$r = \sqrt{3+1} \quad \tan \alpha = \frac{1}{\sqrt{3}}$$

$$r = 2$$

$$\alpha = \frac{\pi}{6}$$

$$\therefore 2 \sin\left(x - \frac{\pi}{6}\right) = \sqrt{3}$$

$$\sin\left(x - \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$x - \frac{\pi}{6} = \frac{\pi}{3}$$

$$x = \pi n + (-1)^n \frac{\pi}{3} + \frac{\pi}{6}$$

where $n \in \mathbb{Z}$

$$c) \int \cos^2 x + \frac{1}{\cos^2 x} dx$$

$$= \int \cos^2 x + \sec^2 x dx$$

$$= \int \frac{1}{2} (\cos 2x + 1) + \sec^2 x dx$$

$$\frac{1}{2} \left(\frac{1}{2} \sin 2x + x \right) + \tan x + c$$

$$d) V = \pi \int_0^{\frac{\pi}{6}} \sin^2 2x dx$$

$$= \frac{\pi}{2} \int_0^{\frac{\pi}{6}} (1 - \cos 4x) dx$$

$$= \frac{\pi}{2} \left[x - \frac{1}{4} \sin 4x \right]_0^{\frac{\pi}{6}}$$

$$= \frac{\pi}{2} \left[\left(\frac{\pi}{6} - \frac{1}{4} \sin \frac{4\pi}{6} \right) - 0 \right]$$

$$= \frac{\pi}{2} \left[\frac{\pi}{6} - \frac{\sqrt{3}}{8} \right]$$

$$= \frac{\pi}{48} [4\pi - 3\sqrt{3}] u^3$$

$$2a) \text{ let } y = \frac{x+1}{x+2}$$

\therefore inverse function is

$$x = \frac{y+1}{y+2}$$

$$xy + 2x = y + 1$$

$$xy - y = 1 - 2x$$

$$y = \frac{1-2x}{x-1}$$

$$\therefore f^{-1}(x) = \frac{1-2x}{x-1}$$

$$b) \cos^{-1}\left(-\frac{1}{2}\right) - \sin^{-1}\left(-\frac{1}{2}\right)$$

$$= \frac{2\pi}{3} + \frac{\pi}{6}$$

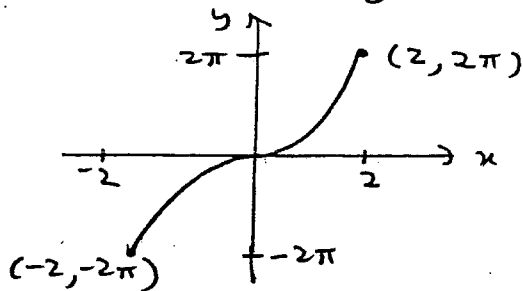
$$= \frac{5\pi}{6}$$

$$c) (i) D: -1 \leq \frac{x}{2} \leq 1$$

$$\therefore -2 \leq x \leq 2$$

$$R: -\frac{\pi}{2} \leq \frac{y}{4} \leq \frac{\pi}{2}$$

$$-2\pi < y \leq 2\pi$$



$$(ii) y = 4 \sin^{-1}\left(\frac{x}{2}\right)$$

$$y' = 4 \cdot \frac{1}{\sqrt{1 - \frac{x^2}{4}}}$$

$$y' = \frac{2}{\sqrt{\frac{4-x^2}{4}}}$$

$$y' = \frac{4}{\sqrt{4-x^2}}$$

$$\text{at } x = 1 \quad y' = \frac{4}{\sqrt{3}}$$

$$\text{at } x = 1 \quad y = 4 \sin^{-1} \frac{1}{2} = \frac{4\pi}{6} = \frac{2\pi}{3}$$

\therefore eqn. of tangent is

$$y - \frac{2\pi}{3} = \frac{4}{\sqrt{3}}(x - 1)$$

$$3\sqrt{3}y - 2\sqrt{3}\pi = 12x - 12$$

$$\therefore 12x - 3\sqrt{3}y + 2\sqrt{3}\pi - 12 = 0$$

$$1) \frac{d}{dx} \left[e^{\tan^{-1}(\ln[\cos 2x])} \right]$$

$$e^{\tan^{-1}(\ln[\cos 2x])} \cdot \frac{1}{1 + [\ln(\cos 2x)]^2} \cdot \frac{1}{\cos 2x} \cdot -2 \sin 2x$$

$$= -2 e^{\tan^{-1}(\ln(\cos 2x))} \cdot \frac{\tan 2x}{1 + [\ln(\cos 2x)]^2}$$

$$1) \text{ LHS} = \int_1^{\sqrt{2}} \frac{dx}{\sqrt{4-2x^2}}$$

$$= \int_1^{\sqrt{2}} \frac{dx}{\sqrt{2(2-x^2)}}$$

$$= \frac{1}{\sqrt{2}} \int_1^{\sqrt{2}} \frac{dx}{\sqrt{2-x^2}}$$

$$= \frac{1}{\sqrt{2}} \left[\sin^{-1} \frac{x}{\sqrt{2}} \right]_1^{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} \left[\sin^{-1} 1 - \sin^{-1} \frac{1}{\sqrt{2}} \right]$$

$$= \frac{1}{\sqrt{2}} \left[\frac{\pi}{2} - \frac{\pi}{4} \right]$$

$$= \frac{\pi}{4\sqrt{2}}$$

$$\text{RHS} = 6 \int_1^{\sqrt{6}} \frac{dx}{\sqrt{2} \sqrt{4+2x^2}}$$

$$= 6 \int_1^{\sqrt{6}} \frac{dx}{\sqrt{2} \cdot 2(2+x^2)}$$

$$= 3 \int_1^{\sqrt{6}} \frac{dx}{\sqrt{2} (2+x^2)}$$

$$= 3 \left[\frac{1}{\sqrt{2}} \tan^{-1} \frac{x}{\sqrt{2}} \right]_1^{\sqrt{6}}$$

$$= \frac{3}{\sqrt{2}} \left(\tan^{-1} \sqrt{3} - \tan^{-1} 1 \right)$$

$$= \frac{3}{\sqrt{2}} \left(\frac{\pi}{3} - \frac{\pi}{4} \right)$$

$$= \frac{3\pi}{12\sqrt{2}}$$

$$= \frac{\pi}{4\sqrt{2}}$$

∴ LHS = RHS