

EXERCISES – Polynomials & BINOMIAL THEOREM I

Name: _____

1. (a) Where does the parabola :
 $y = x^2 + 4x + 10$ cut the y-axis?

(b) Using the method of completing the squares, find its vertex.

(c) What is the minimum value of this function?

2. (a) Find the co-efficient of x^2 in the expansion of :

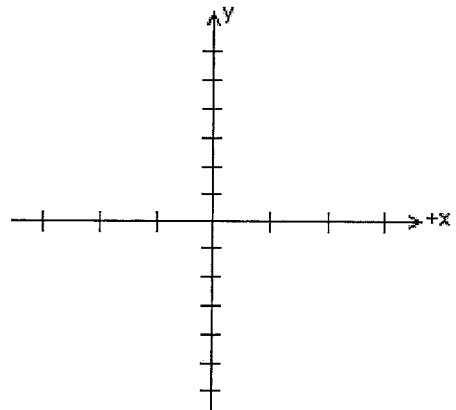
$$(x^2 - 3x - 1)(3x^2 - x + 2)$$

(b) Find **a**, **b** and **c** if :-

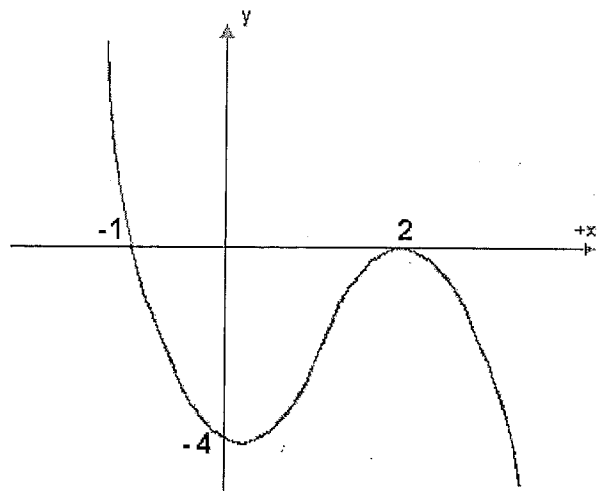
$$ax(x+1) + b(x+1) + c = 2x^2 + x - 1$$

3. (a) Where does the curve :
 $y = (x+1)(2-x)(x+3)(2x-1)$
cut the x and y axes?

(b) Sketch the curve:



4. What is the equation of the curve below?



Equation: _____

5. (a) Show that $(x+3)$ is a factor of :

$$x^3 + x^2 - 9x - 9$$

(b) Find the other factors and fully factorise the above polynomial.

(c) Solve the equation:

$$x^3 + x^2 - 9x - 9 = 0$$

6. (a) What is the co-efficient of x^3 in the following:

(i) $(x+1)^{20}$

(ii) $(1-x)^{15}$

(iii) $(2-x)^{10}$

7. (a) Expand the binomial $(2x+3)^5$

8. (a) Simplify

(i) $\frac{8!}{6!}$ (ii) $\frac{8!}{6!2!}$ (iii) $\frac{(n+1)!}{n!}$

(b) If ${}^nC_3 = an^3 + bn^2 + cn$

Find a, b and c

8. If $x-5$ and $x+2$ are both factors of the polynomial :

$$x^3 - 5x^2 + ax + b$$

Find a and b.

EXERCISES – Polynomials

Name: Amy Ow

1. (a) Where does the parabola :

$y = x^2 + 4x + 10$ cut the y-axis?
when $x = 0$

$y = 10$
cuts y-axis at $y = 10$ ✓

(b) Using the method of completing the squares, find its vertex.

$$\begin{aligned} x^2 + 4x + 10 &= y \\ x^2 + 4x &= y - 10 \\ x^2 + 4x + 4 &= y - 6 \\ (x+2)^2 &= y - 6 \\ \therefore y &= (x+2)^2 + 6 \\ \therefore V \text{ is } &(-2, 6) \end{aligned}$$

(c) What is the minimum value of this function?

concave up
 \therefore min. value is 6 ✓

2. (a) Find the co-efficient of x^2 in the expansion of :

$$\begin{aligned} &(x^2 - 3x - 1) \cdot (3x^2 - x + 2) \\ &= 3x^4 - x^3 + 2x^2 - 9x^3 + 3x^2 - 6x - 3x^2 + x - 2 \\ &= 3x^4 - 10x^3 + 2x^2 - 5x - 2 \\ \therefore \text{co-eff. of } x^2 &\text{ is } 2 \end{aligned}$$

(b) Find a, b and c if :-

$$ax(x+1) + b(x+1) + c = 2x^2 + x - 1$$

$$\begin{aligned} \text{LHS} &= ax^2 + ax + bx + b + c \\ &= ax^2 + (a+b)x + b+c \end{aligned}$$

$$\begin{aligned} \therefore a &= 2 \\ a+b &= 1 \\ \text{sub } a=2 & \\ \therefore b &= -1 \\ b+c &= -1 \\ \text{sub } b=-1 & \\ \therefore c &= 0 \end{aligned}$$

$\therefore a = 2, b = -1, c = 0$ ✓

3. (a) Where does the curve :

$$y = (x+1)(2-x)(x+3)(2x-1)$$

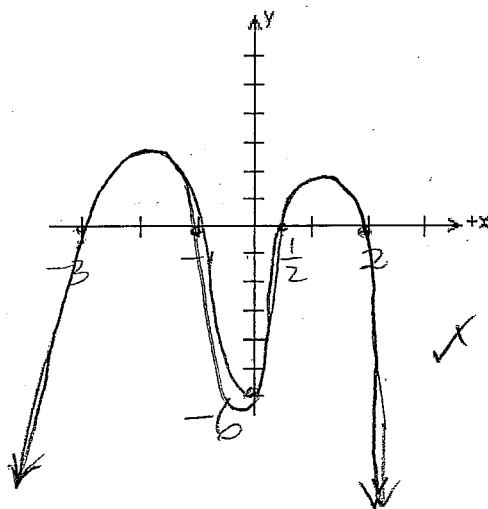
cut the x and y axes?

when $x = 0, y = -6$
y-int is $(0, -6)$ ✓

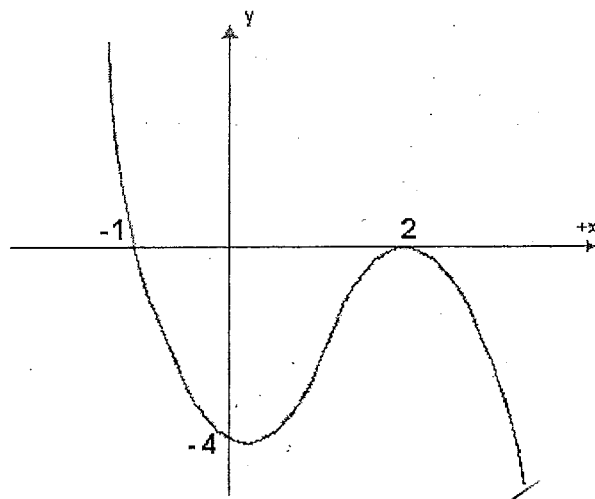
when $y = 0, x = -1, 2, -3, \frac{1}{2}$

x-ints are $x = -3, -1, \frac{1}{2}, 2$ ✓

(b) Sketch the curve:



4. What is the equation of the curve below?



Equation: $k(x-2)^2(x+1) = \dots (i)$

Sub $(0, -4)$ into (i),

$$k(-2)^2(1) = -4$$

$$k = -\frac{4}{4} = -1$$

$$\therefore y = -(x-2)^2(x+1)$$

5. (a) Show that $(x+3)$ is a factor of:

$$x^3 + x^2 - 9x - 9$$

$$\begin{aligned} \text{let } P(x) &= x^3 + x^2 - 9x - 9 \\ P(-3) &= (-3)^3 + (-3)^2 - 9(-3) - 9 \\ &= -27 + 9 + 27 - 9 \\ &= 0 \\ \therefore (x+3) &\text{ is a factor} \end{aligned}$$

(b) Find the other factors and fully factorise the above polynomial.

$$\begin{array}{r} x^2 - 2x - 3 \\ x+3 \overline{) x^3 + x^2 - 9x - 9} \\ \underline{x^3 + 3x^2} \\ -2x^2 - 9x \\ \underline{-2x^2 - 6x} \\ -3x - 9 \\ \underline{-3x - 9} \\ 0 \end{array}$$

$$\therefore P(x) = (x+3)(x+1)(x-3)$$

(c) Solve the equation:

$$x^3 + x^2 - 9x - 9 = 0$$

$$\text{when } P(x) = 0$$

$$x = 3, -1, -3$$

(iii) $(2-x)^{10}$

7. (a) Expand the binomial $(2x+3)^5$

8. (a) Simplify

(i) $\frac{8!}{6!}$ (ii) $\frac{8!}{6!2!}$ (iii) $\frac{(n+1)!}{n!}$

(b) If ${}^nC_3 = an^3 + bn^2 + cn$

Find a, b and c

6. (a) What is the co-efficient of x^3 in the following:

(i) $(x+1)^{20}$

(ii) $(1-x)^{15}$

8. If $x-5$ and $x+2$ are both factors of the polynomial:

$$x^3 - 5x^2 + ax + b = P(x)$$

Find a and b.

$$\begin{aligned} P(5) &= (5)^3 - 5(5)^2 + a(5) + b \\ &= 125 - 125 + 5a + b = 0 \end{aligned}$$

$$\begin{aligned} P(-2) &= (-2)^3 - 5(-2)^2 + a(-2) + b \\ &= -8 - 20 - 2a + b = 0 \end{aligned}$$

$$\therefore \begin{cases} 5a + b = 0 \\ -2a + b = 28 \end{cases}$$

$$\underline{7a = -28}$$

$$a = -4 \quad b = 20$$