

4.1.2 Exercises

Question 1

Solve

i. $\frac{1}{x-1} \leq \frac{1}{x}$

ii. $\frac{x^2-2}{x} \geq 1$

iii. $\frac{2}{x^2} \geq x+1$

Question 2

A geometric series has first term 1, common ratio x and limiting sum S . Find the set of values of x for which $S < 2$.

Question 3

Solve $|2x-3| \leq |x+6|$.

Question 4

i. Solve $|x+3| = 1-x$.

ii. On the same diagram, sketch the graphs of $y = |x+3|$ and $y = 1-x$. Use the graphs to solve $|x+3| > 1-x$.

Question 5

i. Solve $2|x| = x+3$.

ii. On the same diagram, sketch the graphs of $y = 2|x|$ and $y = x+3$. Use the graphs to solve $2|x| \geq x+3$.

Question 6

Solve $|x^2-5| < 4$.

4.1.3 Further Exercises

Question 1

Solve

i. $\frac{3}{x+2} \leq 1$

ii. $\frac{3}{2x+1} \geq 1$

iii. $\frac{x+2}{2x-1} < 1$

iv. $\frac{2x+1}{x-2} < 1$

Question 2

Solve

i. $\frac{x^2-3}{x} < 2$

ii. $\frac{x^2+3}{x} > 4$

iii. $\frac{2-x^2}{x} \geq 1$

iv. $\frac{x^2-4}{x} \geq 3$

Question 3

Solve i. $\frac{-3}{x+1} \leq x+3$

ii. $\frac{4}{x^2} > x-1$

Question 4

Solve i. $\frac{5x}{x^2+4} < 1$

ii. $\frac{9x}{x^2+1} \geq x$

Question 5

Solve $\frac{1}{x^2} > \frac{1}{x-1}$

Question 6

A geometric series has first term 1, common ratio x and limiting sum S . Find the set of values of x for which $S < 1$.

Question 7

Solve

i. $|x| > |3x-4|$

ii. $3|x| \leq |x-4|$

iii. $|x-1| < |3x+5|$

iv. $2|x+1| \geq |x-2|$

Question 8

Solve

i. $|x-1| \geq 2x-3$

ii. $|2x+1| \leq x+3$

iii. $|x+3| < 2x-1$

iv. $|2x-3| > x+1$

Question 9

Solve i. $|x^2-2x| < x+4$

ii. $|x^2-4x+3| \geq x+3$

Question 10

Solve i. $|x^2-4| < 5$

ii. $|x^2-10| \leq 6$

Question 11

Solve i. $||x|-2| \leq 3$

ii. $||x|-2| < 1$

Exercises 4.1.2

1. i. $\frac{1}{x-1} \leq \frac{1}{x}$

$x^2(x-1) \leq x(x-1)^2, \quad x \neq 0, 1$

$x^2(x-1) - x(x-1)^2 \leq 0$

$x(x-1)\{x - (x-1)\} \leq 0$

$\therefore x(x-1) < 0 \quad (x \neq 0, 1)$

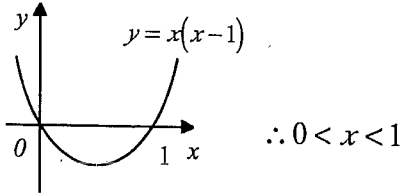


Figure 15.7

ii. $\frac{x^2-2}{x} \geq 1$

$x(x^2-2) \geq x^2, \quad x \neq 0$

$x(x^2-2) - x^2 \geq 0$

$x\{x^2-2-x\} \geq 0$

$x(x-2)(x+1) \geq 0, \quad x \neq 0$

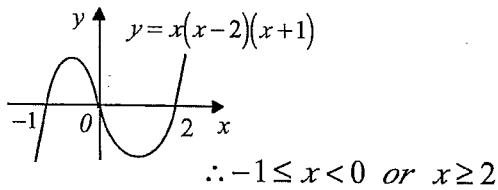


Figure 15.8

iii. $\frac{2}{x^2} \geq x+1$

$2 \geq x^2(x+1), \quad x \neq 0$

$0 \geq x^3 + x^2 - 2$

$0 \geq (x-1)(x^2+2x+2)$

$x^2+2x+2 = (x+1)^2+1 > 0$ for all x

$\therefore x-1 \leq 0, \quad x \neq 0$

$\therefore x < 0$ or $0 < x \leq 1$

2. $|x| < 1$ and $S = \frac{1}{1-x} < 2$.

$1-x < 2(1-x)^2, \quad -1 < x < 1$

$0 < 2(1-x)^2 - (1-x)$

$0 < (1-x)\{2(1-x)-1\}$

$\therefore 0 < (1-x)(1-2x), \quad -1 < x < 1$

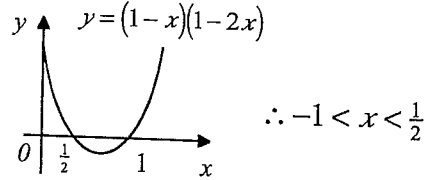


Figure 15.9

3. $|2x-3| \leq |x+6|$
 $(2x-3)^2 \leq (x+6)^2$

$(2x-3)^2 - (x+6)^2 \leq 0$

$(3x+3)(x-9) \leq 0$

$(x+1)(x-9) \leq 0$

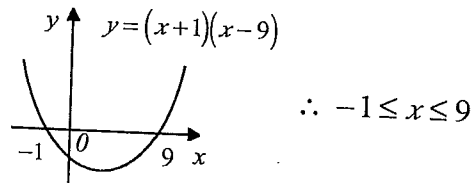


Figure 15.10

4. i. $|x+3| = 1-x$
 $x \leq 1$ and

$x+3 = 1-x$ or $x+3 = -1+x$

$2x = -2$ or no solution

$\therefore x = -1$

ii.

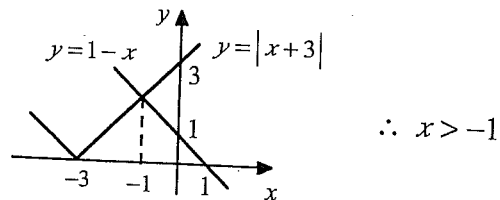


Figure 15.11

5. i. $2|x| = x+3$
 $x \geq -3$ and
 $2x = x+3$ or $2x = -x-3$
 $x = 3$ or $3x = -3$
 $\therefore x = 3$ or $x = -1$

ii.

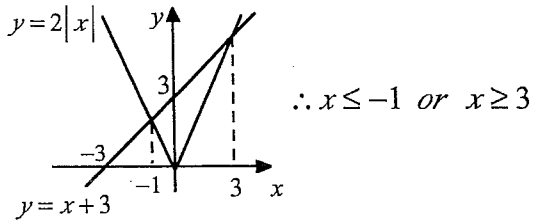


Figure 15.12

6. Consider $|x^2 - 5| = 4$.
 $x^2 - 5 = 4$ or $x^2 - 5 = -4$
 $x^2 = 9$ or $x^2 = 1$

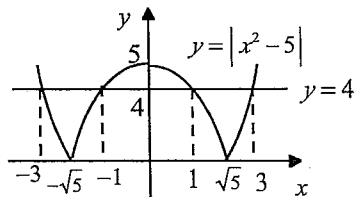


Figure 15.13

$\therefore -3 < x < -1$ or $1 < x < 3$