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MARCELLIN COLLEGE RANDWICK



YEAR 12 HSC

ASSESSMENT TASK # 2

EXTENSION ONE

MATHEMATICS

2005

TOPICS: BINOMIAL THEOREM  
+  
PERMS + COMBS.

Weighting: 10% of H.S.C. Assessment Mark.

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STUDENT NAME: \_\_\_\_\_

MARK: \_\_\_\_\_

PERCENTAGE: \_\_\_\_\_

RANK ON THIS TASK: \_\_\_\_\_

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Time Allowed: 40 minutes.

Directions:

- Answer all questions on separate lined paper.
- Show all necessary working.
- Marks may not be awarded for careless or badly arranged work.

Outcomes examined:

PE3 – Solves problems involving permutations and combinations.

HE3 – Uses a variety of strategies to investigate Mathematical models of situations involving binomial probability.

Question One ( 2 marks )

Write the 5<sup>th</sup> term in the expansion of:  $(a + b)^{11}$

Question Two ( 3 marks )

8 people are to be seated randomly around a round table. What is the probability that a particular husband and wife will not be seated next to each other?

Question Three ( 5 marks )

- How many seven – letter arrangements of the letters of the word COLLEGE are possible?
- In how many of the seven – letter arrangements in a) above will the 2 L's be next to each other?
- How many seven – letter arrangements can be made from the letters of the word COLLEGE if the 2<sup>nd</sup> letter must be a G and the 1<sup>st</sup> letter cannot be an L?

Question Four ( 2 marks )

Find the term independent of x in the expansion of  $(x^2 + \frac{1}{x})^{18}$

Question Five ( 4 marks )

A machine produces electronic components for computers. Sampling shows that the probability of a particular component being faulty is 12%. In a random sample of 16 components, what is the probability that:

- exactly 1 component is faulty? ( Answer correct to 3 decimal places )
- less than 3 components are faulty? ( Answer correct to 3 decimal places )

Question Six ( 5 marks )

- Write down the binomial expansion of  $(1 + x)^n$
- By differentiating  $x^3(1 + x)^n$  and its binomial expansion, show that:

$$\sum_{r=0}^n (r+3) {}^n C_r = (n+6) \cdot 2^{n-1}$$

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SOLUTIONS

MARKING SCHEME

Question One

$$T_{k+1} = {}^n C_k a^{n-k} b^k$$

$$\therefore T_5 = {}^{11} C_4 a^7 b^4$$

$$\text{or } 330 a^7 b^4$$

1 mark for general term

) 1 mark for correct term in either form

Question Two

No. of ways of arranging

8 people in a circle = 5040

1 mark for total number of ways.

Treat husband and wife

as 1  $\therefore$  7 people = 720 ways

But husband & wife could alternate positions.

$\therefore$  Total ways together = 1440

1 mark for total ways together

$\therefore P(\text{not together}) = \frac{5040 - 1440}{5040}$

$$= \frac{5}{7}$$

1 mark for correct probability

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SOLUTIONS

MARKING SCHEME

Question Three

a)  $\frac{7!}{2!2!} = 1260$

1 mark for correct no. of ways.

b) Treat 2 L's as one.

∴ Total number of ways with L's next to each other =

$$\frac{6!}{2!}$$

$$= 360$$

1 mark for correct no. of ways.

c) No. of possible letters

as 1st letter = 3

No. of possible letters as

2nd letter = 1

Remaining 5 letters:

If C or O used as 1st

letter,  $\frac{5!}{2!2!} = 30$

If an E used as 1st letter,

$$\frac{5!}{2!} = 60$$

1 mark for total no. of ways. Letters after the 1st 2 can be arranged.

∴ Total ways =  $3 \times 1 \times 90$

$$= 270.$$

1 mark for multiplying the 3 different restrictions correctly together.

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SOLUTIONS

MARKING SCHEME

Question Four

$$\begin{aligned} T_{k+1} &= {}^{18}C_k (x^2)^{18-k} (x^{-1})^k \\ &= {}^{18}C_k x^{36-2k} x^{-k} \\ &= {}^{18}C_k x^{36-3k} \end{aligned}$$

New term indep. of x

When k = 12

$$\begin{aligned} \therefore \text{Term indep.} \\ \text{of } x &= {}^{18}C_{12} \\ &= 18564 \end{aligned}$$

Question Five

a) P(exactly 1 faulty)

$$\begin{aligned} &= {}^{16}C_1 (0.12)(0.88)^{15} \\ &= 0.282 \text{ (3dp)} \end{aligned}$$

b) P(less than 3 are faulty)

$$\begin{aligned} &= P(\text{No F}) + P(1F) + P(2F) \\ &= {}^{16}C_0 (0.88)^{16} + \text{ANSWER ABOVE} + {}^{16}C_2 (0.12)^2 (0.88)^{14} \\ &= 0.700 \text{ (3dp)} \end{aligned}$$

1 mark for correct general term

1 mark for correct term

1 mark for correct binomial expression

1 mark for correct prob (3dp)

1 mark for correct breakdown of possibilities

1 mark for correct prob. (3dp)

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SOLUTIONS

MARKING SCHEME

Question Six

a)  $(1+x)^n$

$= {}^n C_0 + {}^n C_1 x + {}^n C_2 x^2 + \dots + {}^n C_n x^n$

1 mark for correct expansion

b)  $x^3(1+x)^n = {}^n C_0 x^3 + {}^n C_1 x^4 + {}^n C_2 x^5 + \dots + {}^n C_n x^{n+3}$

Differentiating both sides:

1 mark for this statement

$x^3 n(1+x)^{n-1} + 3x^2(1+x)^n = 3 {}^n C_0 x^2 + 4 {}^n C_1 x^3 + \dots + (n+3) {}^n C_n x^{n+2}$

$x^2(1+x)^{n-1} [xn + 3(1+x)] = \sum_{r=0}^n (r+3) {}^n C_r x^{r+2}$

let  $x=1$

$\therefore 2^{n-1} [n+6] = \sum_{r=0}^n (r+3) {}^n C_r$

1 mark for correct simplification

1 mark for letting  $x=1$  and correctly equating to req'd result.

1 mark for correct differentiation of both sides of equality