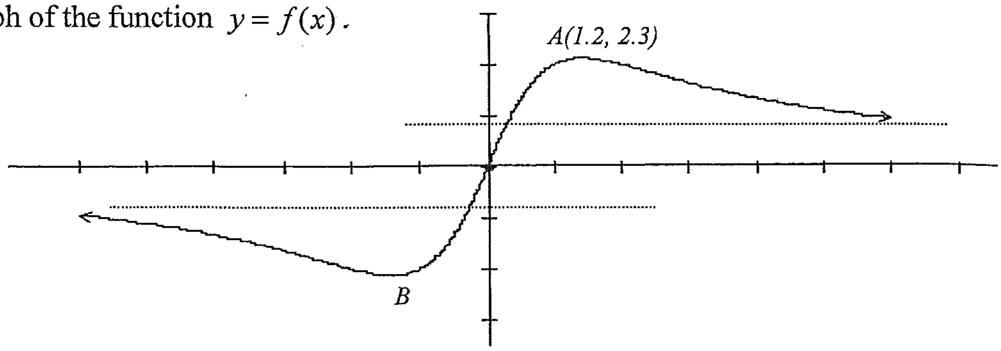
MORIAH COLLEGE – INVERSE FUNCTIONS TEST

Question 1

The following is the graph of the function $y = f(x)$.



i. Explain why the function does not have an inverse function

ii. Given that the point A is a stationary point, and that $y = f(x)$ is an odd function, restrict the domain so that the origin is in it and $y = f(x)$ has an inverse function.

Question 2

i. Let $f(x) = 2 + e^x$. Find the equation of the $f^{-1}(x)$, the inverse function

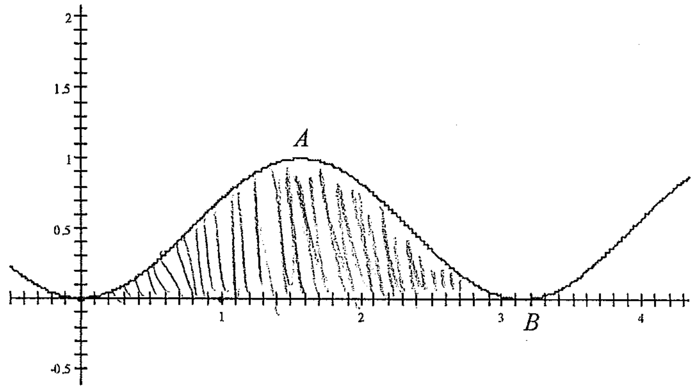
ii. Draw the graphs of $f(x)$ and $f^{-1}(x)$ on the same axes.

Question 3

Sketch the curve of $y = -2\sin^{-1} 3x$, stating clearly the domain and range of the graph

Question 4

Using the substitution $u = 2x^3$, find the value of $\int_0^1 \frac{x^2}{4x^6+1}dx$. Round off your answer to 2 decimal places.



Question 5

The curve shows the graph of the function

$f(x) = \sin^2 x$

i. Find the coordinates of the stationary points A and B

ii. Calculate the shaded area

iii. Let $g(x) = \frac{3}{8}x - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x$

Show that $g'(x) = \sin^4 x$

iv. Using part (iii) or otherwise, find the volume of the body generated by rotating the shaded area around the x-axis. Present your answer in exact form.

Angelia

Moriah College - Inverse Functions Test

Good effort! See corrections!

(i) It doesnt pass the horizontal test. ✓

(ii) $x \geq 1.2$ $\quad -1.2 \leq x \leq 1.2$ (including the origin.

Question 2.

(i) $f(x) = 2 + e^x$

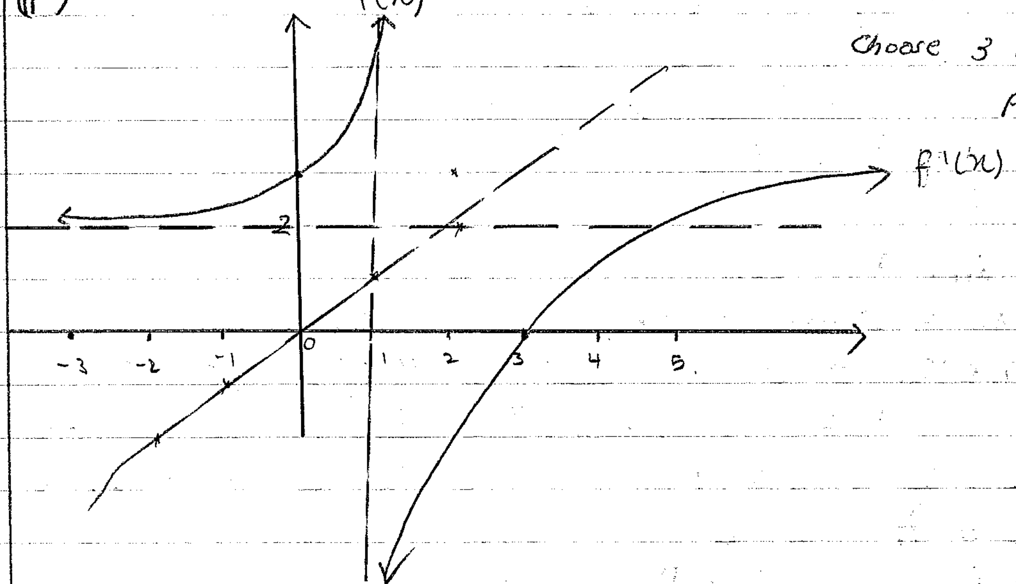
$f^{-1}(x) = 2 + e^y$

$x = 2 + e^y \Rightarrow x - 2 = e^y$

$\log x = 2 + y$ ✗ $\quad \ln(x-2) = \ln e^y$

$y = \ln x - 2$ $\quad \ln(x-2) = y$

(ii)



Choose 3 convenient points, please ask me!

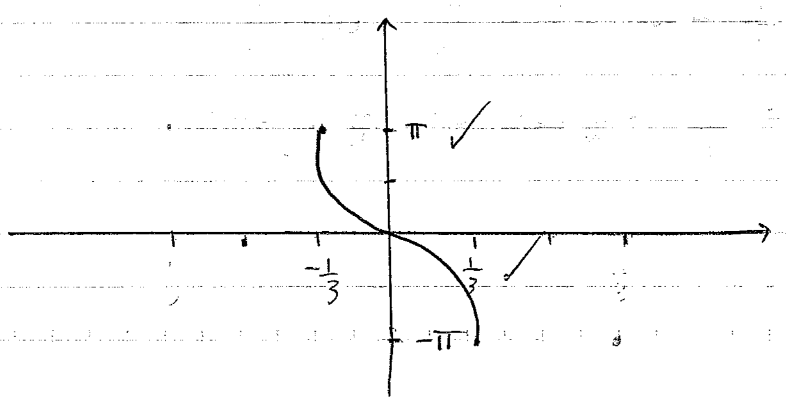
Question 3.

$-1 \leq 3x \leq 1$

$-\frac{1}{3} \leq x \leq \frac{1}{3}$ ✓

Range: $-\frac{\pi}{2} \leq \frac{y}{2} \leq \frac{\pi}{2}$

$-\pi \leq y \leq \pi$ ✓



Question 4.

(a.) Let u be $2x^3$

$$\int_0^1 \frac{x^2}{u^2+1}\,dx.$$

$$\frac{du}{dx} = 6x^2 \checkmark$$

Need to also change the limits to 'u' values
When $x=0$, $u=0$
$x=1$, $u=2$

$$\frac{1}{6}\int_0^2 \frac{6x^2}{u^2+1}\,dx$$

$$\frac{1}{6}\int_1^2 \frac{1}{u^2+1}\,du$$

$$= \left[\frac{1}{6}\tan^{-1} u\right]_0^2$$

$$= \left[\frac{1}{6}\tan^{-1}(2x^3)\right]_0^1$$

$$= \frac{1}{6}\tan^{-1} 2 - \frac{1}{6}\tan^{-1} 0$$

$$= 0.18 \checkmark$$

Question 5

$f(x) = \sin^2 x.$

(i.) $f'(x) = 2\sin x\cos x \checkmark$

$2\sin x\cos x = 0$ or $\sin 2x = 0$ is quicker.

$x = 0, \pi, 2\pi, \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad 2x = 0, \pi, 2\pi, \ldots$

$x = 0, \frac{\pi}{2}, \pi$

If $x = \frac{\pi}{2}$, $y = 1$

$x = \pi$ $y = 0$

A $(\frac{\pi}{2}, 1) \checkmark$ B $(\pi, 0.) \checkmark$

(ii)

$$\int_0^\pi \frac{1}{2} - \frac{1}{2}\cos 2x\,dx \checkmark$$

$$\left[\frac{1}{2}x - \frac{1}{4}\sin 2x\right]_0^\pi \checkmark$$

$$\frac{\pi}{2} - 0 - 0 = \frac{\pi}{2} \text{ units}^2.$$

(iii) $g'(x) = \frac{3}{8} - \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x. \checkmark$

60.II / 80

~~EMOTION 8~~

$$= \frac{3}{8} - \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x$$

$$= \frac{3}{8} - \frac{1}{2}\left(1 - 2\sin^2 \cancel{\phi}_x\right) + \frac{1}{8}\left(1 - 2\sin^2 2x\right)$$

$$= \cancel{\frac{3}{8}} - \cancel{\frac{1}{2}} + \sin^2\cancel{\phi}_x + \cancel{\frac{1}{8}} - \frac{1}{4}\sin^2 2x.$$

$$= \sin^2\cancel{\phi}_x - \frac{1}{4} \times \left(4\cos^2\cancel{\phi}_x \sin^2\cancel{\phi}_x\right)$$

$$= \sin^2\cancel{\phi}^x - \cos^2\cancel{\phi}^x \sin^2\cancel{\phi}^x \quad ✗$$

$$= \sin^2\cancel{\phi}^x\left(1 - \cos^2\cancel{\phi}_x\right) \quad ✓$$

$$= \sin^4\cancel{\phi}_x$$

(iv)

$$\pi\int_0^{\pi} \frac{3}{8}x - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x \, dx.$$

$$\pi\left[\frac{3x^2}{16} + \frac{1}{8}\cos 2x - \frac{1}{128}\cos 4x\right]_0^{\pi} ✓$$

$$= \pi\left[\left(\frac{3\pi^2}{16} + \cancel{\frac{1}{8}} - \cancel{\frac{1}{128}}\right) - \left(0 + \cancel{\frac{1}{8}} - \cancel{\frac{1}{128}}\right)\right]$$

$$= \pi\left[\frac{3\pi^2}{16}\right] ✓$$

$$= \frac{3\pi^3}{16} ✓ \text{units}^3.$$