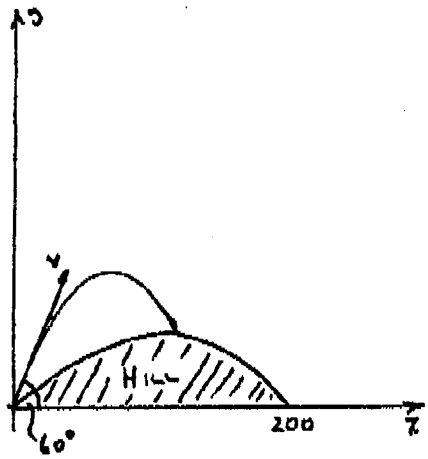
1. When a projectile is fired with speed V at an angle of elevation equal to $60°$ it lands exactly at the top of a hill which is in the shape of a parabola with equation:

$$y = \frac{x(200 - x)}{500},$$

where x and y are measured in metres. In the following assume that air resistance is negligible.



   (a) State the equations of motion and derive x and y as functions of t.

   (b) Eliminate t from these equations in order to find the Cartesian equation of motion.

   (c) Show that the initial velocity of the projectile is given by

$$V^2 = \frac{1000g}{5\sqrt{3} - 1}\,\text{ms}^{-1}.$$

   (d) Suppose now that the projectile is fired at another angle α. Show that if it lands on the hill then the horizontal range is

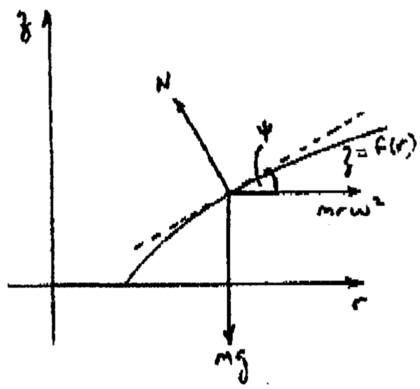
$$x = \frac{(500\tan\alpha - 200)2V^2}{500g\sec^2\alpha - 2V^2}.$$

   (e) Show that this range will be a maximum if

$$\tan^2\alpha - \tfrac{4}{5}\tan\alpha + \left(\frac{2V^2}{500g} - 1\right) = 0.$$

   (f) Hence find the maximum range using the value of V found in part (c). Use the approximation $g = 10\,\text{ms}^{-2}$. Also find the angle α, correct to the nearest minute.

2. A circular curve on an expressway is designed with variable banking so that a motor bike travelling at a constant speed $v\,\text{ms}^{-1}$ and normal to the road surface experiences no sideways frictional force, regardless of how far it is from the centre of the curve. At distance r from the centre of the curve the height of the road surface is z, measurements being in metres.

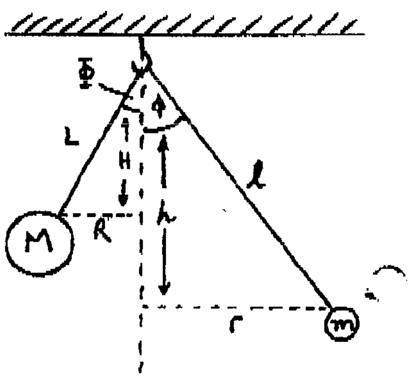


   (a) Let the angle of the banking at r be ψ. Do a force balance and show that $\tan\psi = \dfrac{v^2}{rg}$.

   (b) Hence find z as a function of r, given that $z = 0$ when $r = 1$. [HINT: what is the relationship between $\dfrac{dz}{dr}$ and ψ?]

3. An object falls from rest and experiences forces due to gravity and air resistance. The magnitude of the air resistance is $\dfrac{mv}{10}$ where m and v are the mass and speed of the object respectively. The object reaches the ground in 10 s. From what height did it fall and what percentage of its terminal velocity did it reach? Use the approximation $g = 10\,\text{ms}^{-2}$ and give your answers correct to three significant figures.

4. Two masses M and m are joined by light rods of length L and ℓ respectively to a hook which is free to turn. The tension in both rods is T when the system is turning with angular velocity ω. The mass M is H metres below and R metres to one side of the hook and the mass m is h metres below and r metres on the opposite side of the hook. The angles that the two rods make with the vertical are Φ and ϕ respectively.



   (a) Draw a force diagram for each mass.

   (b) Resolve forces horizontally for each mass and hence show that

$$ML = m\ell.$$

   (c) Resolve forces vertically for each mass and hence show that $H = h$.

   (d) (i) Given that $M = 2m$ and $\phi = 2\Phi$, find the size of these two angles to the nearest minute.

   (ii) Also the shorter rod is 10 cm long. Find the exact angular velocity, and the period of the rotation, correct to two decimal places.

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Ningei Hope you can understand this scribble.

Vert.

$T\cos\Phi = Mg$

$T\cos\phi = mg$

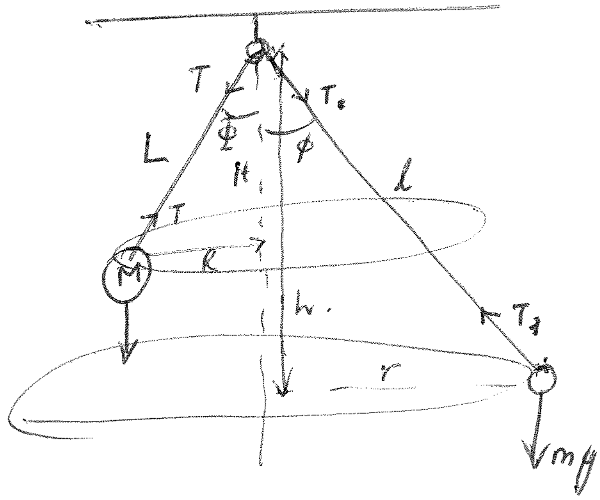
$\cos\Phi = \frac{H}{L}$, $\cos\phi = \frac{h}{l}$

$\therefore \dfrac{\frac{H}{L}}{\frac{h}{l}} = \frac{M\cancel{g}}{m\cancel{g}}$

$\therefore \frac{Hl}{hL} = \frac{M}{m}$

$\therefore \frac{H}{h} ml = ML$

$H = L$



Horizontally

Also $T\sin\Phi = MR\omega^2$

$T.\frac{\cancel{R}}{L} = M.\cancel{R}\omega^2$

$\frac{T}{L} = M\omega^2$

Also $T\sin\phi = mr\omega^2$

$\frac{T\cancel{r}}{l} = m\cancel{r}\omega^2$

$T = ml\omega^2$

$\therefore ML\cancel{\omega^2} = ml\cancel{\omega^2}$ ✓

(d) (i) Since

$\frac{M}{m} = \frac{l}{L}$

$\frac{2\cancel{m}}{\cancel{m}} = \frac{l}{L}$

$l = 2L.$

$\dfrac{\cos\Phi}{\cos\phi} = \frac{M}{m}$

$\therefore \frac{\cos\Phi}{\cos 2\Phi} = \frac{l}{L} = 2$

$\therefore \cos\Phi = \cancel{2(1-2} \quad 2(2\cos^2\Phi - 1)$

$4\cos^2\Phi - 2 - \cos\Phi = 0$

$\therefore \cos\Phi = \frac{1 \pm \sqrt{1-4(4)(-2)}}{8}$

$= \frac{1 \pm \sqrt{33}}{8}$

Acute $\Phi = \frac{1+\sqrt{33}}{8} = 32°32'$ (to nearest) $\phi = 65°4'$

$L = 10$ cm.

$T = ML\omega^2$

But $T\cos\Phi = Mg$

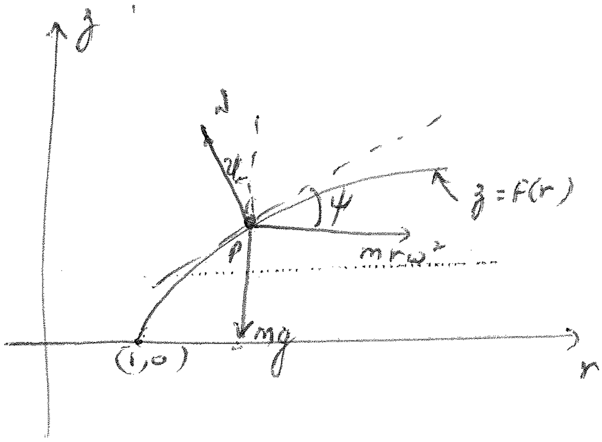
$\therefore \frac{Mg}{\cos\Phi} = ML\omega^2$

$\dfrac{g}{\frac{1+\sqrt{33}}{8}} = 10\omega^2$

$\omega^2 = \frac{g}{0.8431} \times \frac{1}{10}$

$\omega = 1.09$ rad/s.

$v = r\omega \Rightarrow \quad \omega = \frac{v}{r}$



(a)

Vert.

$N\cos\psi = mg \quad \ldots (i)$

Hor

$N\sin\psi = mr\omega^2 \quad \ldots (ii)$

$$\tan\psi = \frac{r\omega^2}{g}$$

$$= \frac{r}{g}\,\frac{v^2}{r^2}$$

$$= \frac{v^2}{rg}$$

(b) $\frac{dz}{dr}$ = grad of tangent at P

$= \tan\psi$

$$\therefore \frac{dz}{dr} = \frac{v^2}{rg}$$

$$\therefore \int \frac{dz}{dr}\,dr = \frac{v^2}{g}\int \frac{1}{r}\,dr$$

$$z = \frac{v^2}{g}\ln r + c$$

At $z = 0, r = 1$

$\therefore c = 0$

$$\therefore z = \frac{v^2}{g}\ln r$$