Test yourself

Revision questions

45. Find the volume of the solid of revolution formed when the curve $y = x^2 - 3$ is rotated about the x-axis from $x = 0$ to $x = 2$.

46. Find the area between the line $y = 3x - 4$, the y-axis, and the lines $y = 0$ and $y = 4$.

47. Find $\int_2^3 (2x - 5)^4 \, dx$.

48. Find the volume of the solid formed when the curve $y = \sqrt{5 - x^2}$ is rotated about the y-axis from $y = 0$ to $y = 2$.

49. Use the substitution $u = ax + b$ to show that:

$$\int (ax + b)^n \, dx = \frac{(ax + b)^{n+1}}{a(n + 1)} + C.$$

50. Evaluate $\int_1^4 \frac{t}{\sqrt{t}} \, dt$.

51. Use Simpson's Rule with 5 function values to find an approximate value of $\int_0^1 (x^3 - 1) \, dx$.

52. Find $\int (2x - 1)(x + 7) \, dx$.

53. If the gradient of a curve is given by $\frac{dy}{dx} = 2x - 1$, and the curve passes through $(3, -4)$, find the equation of the curve.

54. Use the Trapezoidal Rule with 4 subintervals to find an approximation for $\int_3^5 \frac{dx}{x}$.

Challenge questions

55. (a) Find the area enclosed between the line $y = x$ and the curve $y = x^2$.

(b) Find the volume of the solid formed if this area is rotated about the x-axis.

56. Show that $f(x) = x^3 - x$ is an odd function and hence find the value of $\int_{-1}^1 f(x) \, dx$. Find the total area between $f(x)$, the x-axis, and the lines $x = -1$ and $x = 1$.

57. The curve $y = \frac{1}{x + 1}$ is rotated about the x-axis between $x = 1$ and $x = 2$. Use Simpson's Rule with 3 function values to find an approximation of the volume of the solid formed (correct to 3 significant figures).

58. Use the substitution $u^2 = x^5 + 1$ to find

$$\int \frac{x^4}{\sqrt{x^5 + 1}} \, dx.$$

59. Find the integral (primitive function) of $(x + 1)(x^2 + 2x - 5)^5$ by using the substitution $u = x^2 + 2x - 5$.

60. (a) Differentiate $(2x^2 - 3)^4$.

(b) Find $\int_1^2 x(2x^2 - 3)^3 \, dx$.

61. Find the exact volume of the solid formed if the curve $y = \sqrt[4]{3x - 1}$ is rotated about the x-axis from $x = 1$ to $x = 2$.

62. (a) Use the Trapezoidal Rule with 2 subintervals to find an approximation of $\int_1^2 \frac{3}{x + 1} \, dx$.

(b) Show that

$$\int (ax + b)^n \, dx = \frac{(ax + b)^{n+1}}{a(n + 1)} + C$$

does not work for $\int \frac{3}{x + 1} \, dx$.

63. (a) Show that

$$\frac{d}{dx}[x(1 + x)^7] = (8x + 1)(1 + x)^6.$$

(b) Hence find $\int_0^1 (8x + 1)(1 + x)^6 \, dx$.

64. (a) Find the area enclosed between the curve $y = x^2 + 2x - 3$ and the x-axis.

(b) Find the volume of the solid formed if this curve is rotated about the y-axis from $y = 0$ to $y = 1$.

Revision questions

45. $\frac{42\pi}{5}$ units^3

46.
$$y = 3x - 4$$
$$y + 4 = 3x$$
$$\frac{y}{3} + \frac{4}{3} = x$$
$$A = \int_0^4 \left(\frac{y}{3} + \frac{4}{3}\right) dy$$
$$= \left[\frac{y^2}{6} + \frac{4y}{3}\right]_0^4$$
$$= (\tfrac{16}{6} + \tfrac{16}{3}) - (0 + 0)$$
$$= 8 \text{ units}^2$$

47. $\frac{1}{5}$

48.
$$y^2 = 5 - x^2,$$
$$\therefore x^2 = 5 - y^2.$$
$$V = \pi \int_a^b x^2\, dy$$
$$= \pi \int_0^2 (5 - y^2)\, dy$$
$$= \pi \left[5y - \frac{y^3}{3}\right]_0^2$$
$$= \pi[(10 - \tfrac{8}{3}) - (0 - 0)]$$
$$= \frac{22\pi}{3} \text{ units}^3$$

49. $dx = \frac{du}{a}$

50.
$$\int_1^4 \frac{t}{\sqrt{t}}\, dt = \int_1^4 t^{\frac{1}{2}}\, dt$$
$$= \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4$$
$$= \left[\frac{2\sqrt{t^3}}{3}\right]_1^4$$
$$= \frac{2\sqrt{4^3}}{3} - \frac{2\sqrt{1^3}}{3}$$
$$= 4\tfrac{2}{3}$$

51. -0.75

52. $\int (2x^2 + 13x - 7)\, dx = \frac{2x^3}{3} + \frac{13x^2}{2} - 7x + C$

53. $y = x^2 - x - 10$

54.
$$\int_3^5 \frac{1}{x}\, dx \doteq \tfrac{1}{2}(3.5 - 3)[f(3) - f(3.5)]$$
$$+ \tfrac{1}{2}(4 - 3.5)[f(3.5) + f(4)]$$
$$+ \tfrac{1}{2}(4.5 - 4)[f(4) + f(4.5)]$$
$$+ \tfrac{1}{2}(5 - 4.5)[f(4.5) + f(5)]$$
$$= \frac{1}{4}\left(\frac{1}{3} + \frac{1}{3.5}\right) + \frac{1}{4}\left(\frac{1}{3.5} + \frac{1}{4}\right)$$
$$+ \frac{1}{4}\left(\frac{1}{4} + \frac{1}{4.5}\right) + \frac{1}{4}\left(\frac{1}{4.5} + \frac{1}{5}\right)$$
$$= \tfrac{1}{4}(2.049)$$
$$\doteq 0.512$$

or $\int_a^b f(x)\, dx \doteq \frac{h}{2}[(y_0 + y_4) + 2(y_1 + y_2 + y_3)]$,

where $h = \frac{5 - 3}{4} = \frac{1}{2}$.

Challenge questions

55. (a) $\frac{1}{6}$ units^2

(b) $\frac{2\pi}{15}$ units^3

56.
$$f(-x) = (-x)^3 - (-x)$$
$$= -x^3 + x$$
$$= -(x^3 - x)$$
$$= -f(x),$$
$\therefore f(x)$ is an odd function,

so $\int_{-1}^1 f(x)\, dx = 0$

$$2\int_0^1 (x^3 - x)\, dx$$
$$= 2\left[\frac{x^4}{4} - \frac{x^2}{2}\right]_0^1$$
$$= 2\left[\left(\frac{1^4}{4} - \frac{1^2}{2}\right) - (0 - 0)\right]$$
$$= 2(\tfrac{1}{4} - \tfrac{1}{2})$$
$$= -\tfrac{1}{2}.$$
$\therefore$ area $= \frac{1}{2}$ unit^2.

57. 0.524 units^3

58. $u^2 = x^5 + 1$.

Then $2u\frac{du}{dx} = 5x^4$ by implicit differentiation,

or $u = (x^5 - 1)^{\frac{1}{2}}$
$$\frac{du}{dx} = \tfrac{1}{2}(x^5 - 1)^{-\frac{1}{2}}(5x^4)$$
$$= \frac{5x^4}{2\sqrt{x^5 + 1}}$$
$$du = \frac{5x^4}{2\sqrt{x^5 - 1}}\, dx$$
$$\int \frac{x^4}{\sqrt{x^5 + 1}}\, dx$$
$$= \tfrac{2}{5}\int \frac{5x^4}{2\sqrt{x^5 - 1}}\, dx$$
$$= \tfrac{2}{5}\int du$$
$$= \tfrac{2}{5}u + C$$
$$= \tfrac{2}{5}\sqrt{x^5 + 1} + C$$

59. $\frac{(x^2 + 2x - 5)^6}{12} + C$

60. (a) $\frac{dy}{dx} = 4(2x^2 - 3)^3 \times 4x$

$= 16x(2x^2 - 3)^3$

(b) $\int_1^2 x(2x^2 - 3)^3\,dx$

$= \frac{1}{16}\int_1^2 16x(2x^2 - 3)^3\,dx$

$= \frac{1}{16}\left[(2x^2 - 3)^4\right]_1^2$

$= \frac{1}{16}[5^4 - (-1)^4]$

$= \frac{1}{16}[625 - 1]$

$= 39$

61. $\frac{2\pi}{9}(\sqrt{125} - \sqrt{8}) = \frac{2\pi}{9}(5\sqrt{5} - 2\sqrt{2})$ units^3

62. (a) $\int_1^{1.5} \frac{3}{x+1}\,dx + \int_{1.5}^2 \frac{3}{x+1}\,dx$

$= \frac{1.5 - 1}{2}[f(1) + f(1.5)]$

$+ \frac{2 - 1.5}{2}[f(1.5) + f(2)]$

$= \frac{0.5}{2}\left[\frac{3}{1+1} + \frac{3}{1.5+1}\right]$

$+ \frac{0.5}{2}\left[\frac{3}{1.5+1} + \frac{3}{2+1}\right]$

$= 1.225$

(b) $\int \frac{3}{x+1}\,dx = \int 3(x+1)^{-1}\,dx$

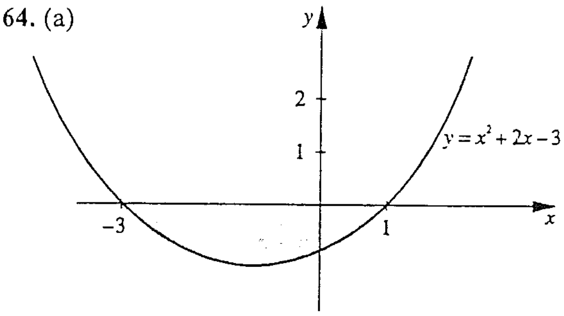
$= \frac{3(x+1)^0}{0} + C$

$\therefore$ this rule does not work.

63. (a) Use product rule.

(b) 128

64. (a)



$\int_{-3}^{1} (x^2 + 2x - 3)\,dx$

$= \left[\frac{x^3}{3} + x^2 - 3x\right]_{-3}^{1}$

$= (\frac{1}{3} + 1 - 3) - (-9 + 9 + 9)$

$= -10\frac{2}{3}$

$\therefore$ area is $10\frac{2}{3}$ units^2.