CRANBROOK SCHOOL

YEAR 11 MATHEMATICS – 3 UNIT TEST

5TH November TERM 4 2003 MJB/HRK/CJL

INDUCTION / FINANCIAL APPLICATIONS OF SERIES Time:45mins

All necessary working should be shown in every question.
Full marks may not be awarded if work is careless or badly arranged.
Approved silent calculators may be used.
Begin each question on a new page.

1. (12marks) (Begin a new page) **HRK**

(a) Use the Principle of Mathematical Induction to show that $9^{n+2} - 4^n$ is divisible by 5 for all positive integers n

(b) Prove by mathematical induction that for $n \geq 1$,

$$1^2 + 3^2 + \ldots + (2n-1)^2 = \frac{1}{3}n(2n-1)(2n+1)$$

2 (12marks) (Begin a new page) **CJL**

On 1 July 2001, Fraser invested $10 000 in a bank account that paid interest at a fixed rate of 9% p.a., compounded annually.

(a) How much would be in the account after the payment of interest on 1 July 2011 if no additional deposits were made?

(b) In fact, Fraser added $1 000 to his account on 1 July each year, beginning on 1 July 2002. How much was in his account on 1 July 2011 after the payment of interest and his deposit?

(c) Fraser's friend, Antony, invested $10 000 in an account at another bank on 1 July 2001 and made no further deposits. On 1 July 2011, the balance of Antony's account was $35 478. What was the annual rate of compound interest paid on Antony's account?

3. (12marks) (Begin a new page) **MJB**

What is the monthly instalment necessary to pay back a personal loan of $15 000 at a rate of $13\frac{1}{2}$% per annum over five years? Give your answer correct to the nearest dollar.

Question 1

(a) 1/ PROVE TRUE FOR $n=1$

$9^{1+2} - 4^1 = 725$

$= 5(145)$

ie is divisible by 5 ✓

2/ Assume true for $n=k$

ie $9^{k+2} - 4^k = 5Q \quad (Q \in J)$

ie $9^{k+2} = 5Q + 4^k$ ✓

3/ Prove true for $n=k+1$

$9^{k+3} - 4^{k+1} = 5R \quad (R \in J)$ ✓

$9(9^{k+2}) - 4(4^k)$

$= 9(5Q + 4^k) - 4(4^k)$ using assumption

$= 9(5Q) + 9(4^k) - 4(4^k)$ ✓

$= 9(5Q) + 5(4^k)$ ✓

$= 5(9Q + 4^k)$

which is divisible by 5/

∴ IF TRUE FOR $n=k$ then statement is true for ✓ $n=k+1$.

But it is true for $n=1$

∴ it is true for $n=1+1=2$

& since true for $n=2$, it is true for $n=3$ and so on

Hence $9^{n+2} - 4^n$ is divisible by 5 for $n \in J^+$

- proven by MI. //

*

(b) 1/ Prove true for $n=1$

LHS $= (2-1)^2$ RHS $= \frac{1}{3}(1$

$= 1$ $= 1$

$= LHS$

∴ True for $n=1$ ✓

2/ Assume true for $n=k$

ie $1^2 + 3^2 + \ldots + (2k-1)^2 = \frac{1}{3}k(2k-1)(2k+1)$ ✓

3/ Prove true for $n=k+1$

ie $1^2 + 3^2 + \ldots + (2k-1)^2 + (2(k+1)-1)^2$ ✓

$= \frac{1}{3}(k+1)(2(k+1)-1)(2(k+1)+1)$

LHS

$= \frac{1}{3}k(2k-1)(2k+1) + (2k+1)^2$

$= \frac{1}{3}(2k+1)\left[k(2k-1) + 3(2k+1)\right]$

$= \frac{1}{3}(2k+1)(2k^2+5k+3)$ ✓

$= \frac{1}{3}(2k+1)(k+1)(2k+3)$

RHS $= \frac{1}{3}(k+1)(2k+1)(2k+3)$ ✓

$= LHS$

∴ IF TRUE FOR $n=k$ statement is true for $n=k+1$ ∴ ∴ ∴ ✓

*

∴ Statement has been proven true by MI //

Question 2

(3)

a) $10\,000(1.09)$

$= \$23\,673.64$ ✓✓

b) let A_n be amount in account after interest

$A_1 = 10\,000(1.09) + 1000$ ✓✓

$A_2 = 10\,000(1.09)^2 + 1000(1.09) + 1000$ ✓✓

$\vdots$

$A_{10} = 10\,000(1.09)^{10} + 1000\left[1.09^9 + \dots + 1\right]$ ✓

G.P. $a = 1$, $r = 1.09$, $n = 10$

$S_{10} = \dfrac{1(1.09^{10} - 1)}{0.09}$ ✓

$= 15.1929\dots$

$= 10\,000(1.09)^{10} + 1000\,[15.1929\dots]$

$= \$38\,866.57$ ✓

c) $35\,478 = 10\,000(1+r)^{10}$ ✓

$3.5478 = (1+r)^{10}$

$1 + r = \sqrt[10]{3.5478}$ ✓

$1 + r = 1.135\dots$

$r = 0.135\dots$

∴ Interest rate is 13.5% p.a. ✓