INTEGRATION BY SUBSTITUTION

1. Using the substitution given, find the following indefinite integrals

a) $\int 3x^2 (x^3 - 1)^4 dx$, $\qquad u = x^3 - 1$

b) $\int x\sqrt{1+x^2} dx$, $\qquad u = 1 + x^2$

c) $\int x\sqrt{1-x} dx$, $\qquad u = 1 - x$

d) $\int \frac{\log_e x}{x} dx$, $\qquad u = \log_e x$

e) $\int \frac{dx}{x(1+\log_e x)^2}$, $\qquad u = 1 + \log_e x$

f) $\int \frac{x}{x+1} dx$, $\qquad x = u^2 - 1$

g) $\int x\sqrt{3x-1} dx$, $\qquad u = \sqrt{3x-1}$

h) $\int x(1-x)^{10} dx$, $\qquad x = u^2 + 1$

2. Using the given substitution, find the following definite integrals

a) $\int_{-1}^{2} x\sqrt{2-x} dx$, $\qquad u = 2 - x$

b) $\int_{0}^{1} \frac{2x}{1+x^2} dx$, $\qquad u = 1 + x^2$

c) $\int_{0}^{\ln 2} \frac{e^x dx}{1+e^x}$, $\qquad u = e^x$

d) $\int_{e}^{e^2} \frac{dx}{x \log_e x}$, $\qquad u = \log_e x$

e) $\int_{0}^{2} \frac{1-x}{x^2 - 2x + 6} dx$, $\qquad u = x^2 - 2x + 6$

f) $\int_{0}^{1} \frac{(1+x)e^x}{1+xe^x} dx$, $\qquad u = xe^x$

g) $\int_{1}^{e} \frac{(1+\ln x)^3}{x} dx$, $\qquad u = 1 + \ln x$

ANSWERS

1.

a) $\dfrac{1}{5}(x^3 - 1)^5 + C$

b) $\dfrac{1}{3}(1 + x^2)^{\frac{3}{2}} + C$

c) $\dfrac{2}{5}(1 - x)^{\frac{5}{2}} - \dfrac{2}{3}(1 - x)^{\frac{3}{2}} + C$

d) $\dfrac{1}{2}(\log_e x)^2 + C$

e) $\dfrac{-1}{1 + \log_e x} + C$

f) $(x + 1) - \log_e |x + 1| + C$

g) $\dfrac{2}{45}(3x - 1)^{\frac{5}{2}} + \dfrac{2}{27}(3x - 1)^{\frac{3}{2}} + C$

h) $\dfrac{(x-1)^{12}}{12} + \dfrac{(x-1)^{11}}{11} + C$

2.

a) $\dfrac{2\sqrt{3}}{5}$ b) $\ln 2$ c) $\ln\left(\dfrac{3}{2}\right)$ d) $\ln 2$

e) 0 f) $\ln(1 + e)$ g) $\dfrac{15}{4}$