DIFFERENTIATION AND INTEGRATION

Instructions

(1) Draw up 3 columns and label them

$f(x)$ $\qquad\qquad$ $f'(x)$ $\qquad\qquad$ $\int f(x)dx$

(2) Copy the question out under an $f(x)$ column and number the question.

(3) Under the $f'(x)$ column, differentiate $f(x)$.

(4) Under the $\int f(x)dx$ column, integrate $f(x)$.

(5) Show all necessary working.

(6) Rule a line across at the end of each question.

Questions (These refer to $f(x)$)

1. x

2. 7

3. $\sqrt[5]{x^6}$

4. $\sqrt{x}$

5. 0

6. $5x^{-1/5}$

7. $6x^2 - 7x + 3$

8. $\dfrac{8}{x^2}$

9. $\dfrac{1}{x}$

10. $\dfrac{1}{3x}$

11. e^x

12. e^{-x}

13. $(2x+7)^7$

14. $\dfrac{1}{4-5x}$

15. $\dfrac{1}{(7x+5)^4}$

16. $\dfrac{1}{9x-2}$

17. $\dfrac{1}{\sqrt{7x-3}}$

18. e^{4x+3}

19. $\sqrt{4x+3}$

20. $\dfrac{5x^4}{x^5+1}$

21. $3x^2(x^3+1)^6$

22. $\dfrac{e^{2x}}{e^{2x}+3}$

23. $(e^x - e^{-x})(e^x + e^{-x})^7$

24. $\dfrac{x+1}{(x^2+2x)^3}$

Miscellaneous Differentiation

Differentiate:

1. $2e^{-x^2}$

2. $\log_e\left(2x^2+5\right)$

3. $\left(3x^2+2x\right)^7$

4. $\left(3x^2+1\right)\left(4x+3\right)^5$

5. $x^2-2\sqrt{x}$

6. $\log_e 2\sqrt{x}$

7. ex^3

8. $\dfrac{x^2}{x-2}$

9. $\log_e e^{2x^2}$

10. $\ln[x(x+1)]$

11. $e^{x^3-2x^2}$

12. $\dfrac{x^2+3x-4}{x}$

13. $\log_e \dfrac{1-x^2}{1+x^2}$

14. $\dfrac{1}{\sqrt{x-2}}$

15. $\left(x+\dfrac{1}{x}\right)^3$

16. $\left(x^2-1\right)^{3/2}\left(x^2+3x\right)^4$

17. $x^2 e^x$

18. x^n+x^{2n}

19. $\left(\sqrt{x}+2\right)^9$

20. $\dfrac{\sqrt{1-x}}{1+x}$

21. $\ln\sqrt{x-1}$

22. $x^2\log_e x$

23. $\sqrt{\dfrac{1+x}{1-x}}$

24. $\left(1+x\right)^3 e^{2x}$

25. $\left(x^3-1\right)\log_e x$

26. $\dfrac{e^{-2x}-e^x}{e^x}$

27. $\log_e \sqrt{\dfrac{1+x}{1-x}}$

28. 2^x

Miscellaneous Integration

1.   (i)   Find the derivative of $\log_e\left(x^2-5x+7\right)$ and hence evaluate

$$\int_3^2 \frac{2x-5}{x^2-5x+7}\,dx$$

(ii)   Obtain $\dfrac{d}{dx}\left(e^x\log_e x\right)$ and hence evaluate $\displaystyle\int_1^2 e^x\left(\frac{1}{x}+\log_e x\right)dx$ giving the

result in irrational form.

2. (i) Differentiate $\log_e(\log_e x)$, and evaluate $\displaystyle\int_e^{e^2} \frac{dx}{x\log_e x}$.

(ii) Find $\dfrac{d}{dx}(\log_e x)^2$, and hence evaluate $\displaystyle\int_1^e \frac{\log_e x}{x}dx$.

3. (i) Find $\dfrac{d}{dx}\left(e^{x^2}\right)$ and hence evaluate $\displaystyle\int_0^1 xe^{x^2}dx$, giving the result correct to 2 decimal places.

(ii) Obtain $\dfrac{d}{dx}\left(xe^x - e^x\right)$ and hence calculate $\displaystyle\int_0^1 xe^x dx$.

answer in exact form.

4. Find $\displaystyle\int\frac{x^2 dx}{\sqrt{x^3 - 1}}$

(a) Using the substitution $u = x^3 - 1$,

(b) Using the substitution $u = \sqrt{x^3 - 1}$.

5. Evaluate $\displaystyle\int_e^{e^2} \frac{dx}{x\ln x}$ using the substitution $u = \ln x$.

6. Shade the area represented by the integral $\displaystyle\int_0^1 \sqrt{1 - x^2}dx$ and hence evaluate the integral.

7. If $\displaystyle\int_{\frac{1}{a}}^a \frac{1}{t^2}dt = \frac{1}{\sqrt{2}}$ and $a > 0$ find a.

8. Show that $\dfrac{x+5}{x+3} = 1 + \dfrac{2}{x+3}$ and hence evaluate $\displaystyle\int_0^1 \frac{x+5}{x+3}dx$ exactly.

9. Use the standard integral $\displaystyle\int\frac{dx}{\sqrt{x^2 + a^2}} = \ln(x + \sqrt{x^2 + a^2}) + c$ to evaluate

$\displaystyle\int_{-1}^1 \frac{3}{\sqrt{2x^2 + 2}}dx$

10. Differentiate $x^3 \ln x$ and hence find an indefinite integral for $x^2 \ln x$

Differentiation and Integration

	$f'(x)$	$\int f(x)\,dx$
1.	1	$\dfrac{x^2}{2}+k$
2.	0	$7x+k$
3.	$\dfrac{6}{5}x^{1/5}$	$\dfrac{5}{11}x^{11/5}+k$
4.	$\dfrac{1}{2\sqrt{x}}$	$\dfrac{2}{3}x^{3/2}+k$
5.	0	k
6.	$-x^{-6/5}$	$\dfrac{25}{4}x^{4/5}+k$
7.	$12x-7$	$2x^3-\dfrac{7}{2}x^2+3x+k$
8.	$\dfrac{-16}{x^3}$	$-8x^{-1}+k$
9.	$-\dfrac{1}{x^2}$	$\log_e|x|+k$
10.	$-\dfrac{1}{3x^2}$	$\dfrac{1}{3}\log_e|x|+k$
11.	e^x	e^x+k
12.	$-e^{-x}$	$-e^{-x}+k$
13.	$14(2x+7)^6$	$\dfrac{1}{16}(2x+7)^8+k$
14.	$\dfrac{5}{(4-5x)^2}$	$-\dfrac{1}{5}\log_e|4-5x|+k$
15.	$-\dfrac{28}{(7x+5)^5}$	$-\dfrac{1}{21(7x+5)^3}+k$
16.	$-\dfrac{9}{(9x-2)^2}$	$\dfrac{1}{9}\log_e|9x-2|+k$
17.	$\dfrac{-7}{2(7x-3)\sqrt{7x-3}}$	$\dfrac{2}{7}\sqrt{7x-3}+k$
18.	$4e^{4x+3}$	$\dfrac{1}{4}e^{4x+3}+k$
19.	$\dfrac{2}{\sqrt{4x+3}}$	$\dfrac{1}{6}(4x+3)\sqrt{4x+3}+k$

Answers

20. $$\dfrac{20x^3 - 5x^8}{\left(x^5+1\right)^2} \qquad\qquad \log_e\left|x^5+1\right|+k$$

21. $$6x\left(10x^3+1\right)\left(x^3+1\right)^5 \qquad \dfrac{1}{7}\left(x^3+1\right)^7+k$$

22. $$\dfrac{6e^{2x}}{\left(e^{2x}+3\right)^2} \qquad\qquad \dfrac{1}{2}\log_e\left(e^{2x}+3\right)+k$$

23. $$\left(e^x+e^{-x}\right)^6\left(8e^{2x}-12+8e^{-2x}\right) \quad \dfrac{1}{8}\left(e^x+e^{-x}\right)^8+k$$

24. $$\dfrac{-\left(5x^2+10x+6\right)}{x^4\left(x+2\right)^4} \qquad \dfrac{-1}{4x^2\left(x+2\right)^2}+k$$

Miscellaneous Differentiation (Answers)

1. $-4xe^{-x^2}$

2. $\dfrac{4x}{2x^2+5}$

3. $14x^6\left(3x+1\right)\left(3x+2\right)^6$

4. $2\left(4x+3\right)^4\left(42x^2+9x+10\right)$

5. $2x-\dfrac{1}{\sqrt{x}}$

6. $\dfrac{1}{2x}$

7. $3ex^2$

8. $\dfrac{x\left(x-4\right)}{\left(x-2\right)^2}$

9. $4x$

10. $\dfrac{1}{x}+\dfrac{1}{x+1}$

11. $x\left(3x-4\right)e^{x^2\left(x-2\right)}$

12. $1+\dfrac{4}{x^2}$

13. $\dfrac{-4x}{\left(1-x^2\right)\left(1+x^2\right)}$

14. $-\dfrac{1}{2\left(x-2\right)^{\frac{3}{2}}}$

15. $3\left(x+\dfrac{1}{x}\right)^2\left(1-\dfrac{1}{x^2}\right)$

16. $\sqrt{x^2-1}\left(x^2+3x\right)^3\left(11x^3+21x^2-8x-12\right)$

17. $xe^x\left(2+x\right)$

18. $nx^{n-1}\left(1+2x^n\right)$

19. $\dfrac{9}{2\sqrt{x}}\left(\sqrt{x}+2\right)^8$

20. $\dfrac{x-3}{2\sqrt{1-x}\left(1+x\right)^2}$

21. $\dfrac{1}{2\left(x-1\right)}$

22. $x\left(1+2\log_e x\right)$

23. $\dfrac{1}{\sqrt{\left(1+x\right)\left(1-x\right)^3}}$

24. $\left(1+x\right)^2 e^{2x}\left(5+2x\right)$

25. $x^2\left(3\log_e x+1\right)-\dfrac{1}{x}$

26. $-3e^{-3x}$

27. $\dfrac{1}{1-x^2}$

28. $2^x\ln 2$

Miscellaneous Integration Answers

1.   (i)   0   (ii) $e^x\left(\dfrac{1}{x}+\log_e x\right)$; $e^2\log_e 2$

2.   (i) $\dfrac{1}{x\log_e x}$; $\log_e 2 \approx 0.6931$ (ii) $\dfrac{2\log_e x}{x}$; $\dfrac{1}{2}$

3.   (i) $2xe^{x^2}$; $\dfrac{1}{2}(e-1) \approx 0.86$ (ii) xe^x;   1

4. $\dfrac{2}{3}\sqrt{x^3-1}+c$

5. $\ln 2$

6. $\dfrac{\pi}{4}$

7. $a=\sqrt{2}$

8. $1+2\ln\dfrac{4}{3}$

9. $\dfrac{3}{\sqrt{2}}\ln(\dfrac{\sqrt{2}+1}{\sqrt{2}-1})$

10. $\dfrac{1}{3}(x^3\ln x-\dfrac{x^3}{3})$