1. Use the principle of mathematical induction to prove the following for all positive integers n:

   (a) $2+5+8+\cdots+(3n-1)=\frac{n}{2}(3n+1)$,

   (b) $3^n \geq 1+2n$,

   (c) $3^{3n}+2^{n+2}$ is divisible by 5.

2. Use the definition $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ to differentiate $f(x)=1-4x-4x^2$.

3. (a) Use the definition $f'(c)=\lim_{x\to c}\frac{f(x)-f(c)}{x-c}$ to show that the derivative of x^2+6x is $2x+6$.

   (b) Hence

      (i) find the gradient of the tangent when $x=-1$,

      (ii) find the point on the curve where the tangent is horizontal,

      (iii) find the point on the curve where the tangent is parallel to the line $4x-y=1$.

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1a. Prove (S) true for $n=1$.

$LHS = 2 \quad RHS = \frac{1}{2}(3+1) = 2$

Since $LHS = RHS$, (S) true for $n=1$.

Suppose (S) true for $n=k$ $(k \in J^+)$

thus: $2+5+8+\dots+(3k-1) = \frac{k}{2}(3k+1) \dots (1)$

Prove that (S) true for $n=k+1$

Prove: $2+5+8+\dots+(3k-1)+(3(k+1)-1) = \frac{k+1}{2}(3(k+1)+1) = \frac{3k^2}{}$

$LHS = 2+5+8+\dots+(3k-1)+(3k+2)$

$= \frac{k}{2}(3k+1) + (3k+2)$ (from (1))

$= \frac{3k^2+k}{2} + 3k+2$

$= \frac{3k^2+k+6k+4}{2}$

$= \frac{3k^2+7k+4}{2}$

$= RHS$.

Conclusion: Since (S) true for $n=1$ and true for $n=k+1$, when true for $n=k$ $(k \in J^+)$, (S) is true for all n $(n \in$

b. Prove (S) true for $n=1$.

$LHS = 3$, $RHS = 1+2 = 3$

Since $LHS \geq RHS$, (S) true for $n=1$.

Suppose (S) true for $n=k$ $(k \in J^+)$

$3^k \geq 1+2k \dots (1)$

Prove true for $n=k+1$

Prove: $3^{k+1} \geq 1+2(k+1)$

$3^{k+1} \geq 1+2k+2$

$3^{k+1} \geq 2k+3$

$LHS = 3^{k+1} = 3^k \times 3$

$\geq (1+2k) \times 3$ (from (1))

$\geq 3+6k$

$\geq (2k+3)+(4k)$

Since $k \geq 1$, $4k > 0$, $LHS \geq 2k+3$

Conclusion

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c. When $n=1$, $3^{3n}+2^{n+2}=3^3+2^3=35$, which is divisible by 5.

Suppose that $3^{3k}+2^{k+2}$ is divisible by 5 $(k\in J^+)$

$3^{3k}+2^{k+2}=5M \quad (M\in J^+) \qquad 2^{k+2}=5M-3^{3k}$

$3^{3k}=5M-2^{k+2} \ldots (1) \qquad 2^k=\frac{5M-3^{3k}}{4} \ldots (2)$

Prove that $3^{3(k+1)}+2^{(k+1)+2}$ is divisible by 5 $(k\in J^+)$

Now, $3^{3k+3}+2^{k+4}=3^{3k}\times 3^3+2^k\times 2^4$

$=3^{3k}\times 27+2^k\times 16 \ldots$ (from (1))

$=(5M-2^{k+2})\times 27+(2^k\times 16)$

$=135M-[(2^k\times 2^2)\times 27]+\left(\frac{5M-3^{3k}}{4}\times 16\right)$ (from (2))

$=135M-[2^k\times 108]+(20M-4(3^{3k}))$

$=155M-(2^k\times 108)-4(3^{3k})$

$=5\left(31M-\frac{2^k\times 108}{5}-\frac{4}{5}(3^{3k})\right)$

which is divisible by 5.

2. $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=\lim_{h\to 0}\frac{1-4(x+h)-4(x+h)^2-1+4x+4x^2}{h}$

$=\lim_{h\to 0}\frac{1-4x-4h-4(x^2+2xh+h^2)-1+4x+4x^2}{h}$

$=\lim_{h\to 0}\frac{-4h-4x^2-8xh-4h^2+4x^2}{h}$

$=\lim_{h\to 0}\frac{-4h^2-8xh-4h}{h}=\lim_{h\to 0}-4h-8x-4$

$f'(x)=-8x-4$

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3a. $f(x)=x^2+6x$

$f'(x)=\lim_{x\to c}\frac{f(x)-f(c)}{x-c}=\lim_{x\to c}\frac{x^2+6x-c^2-6c}{x-c}$

$=\lim_{x\to c}\frac{x^2-c^2}{x-c}$

$=\lim_{x\to c}\frac{(x+c)(x-c)}{(x-c)}$

$=\lim_{x\to c}x+c$

$f'(x)=2c.$

b(i) $x=c=-1$

$f'(-1)=2\times -1$

$=-2$

(ii) $f'(x)=2c=0$

$2c=0$

$c=0.$

$f'(x)=2c$

$y=0.$

$\therefore$ pt $=(0,0)$.

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(iii) $4x-y=1$

$-y=1-4x$

$f'(x)=2c=4.$

$2c=4$

1c. $3^{3k+3} + 2^{k+3}$

$= 3^3 \times 3^k + 2 \times 2^{k+2}$

$= 27(5M - 2^{k+2}) + 2 \times 2^{k+2}$

$= 135M - 27 \times 2^{k+2} + 2 \times 2^{k+2}$

$= 135M - 25 \times 2^{k+2}$

$= 5(27M - 5 \times 2^{k+2})$

which is divisible by 5.

3a. $f(x) = x^2 + 6x$

$$f'(x) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c} = \lim_{x \to c} \frac{x^2 + 6x - c^2 - 6c}{x - c}$$

$$= \lim_{x \to c} \frac{x^2 - c^2 + 6x - 6c}{x - c}$$

$$= \lim_{x \to c} \frac{(x-c)(x+c) + 6(x-c)}{x - c}$$

$$= \lim_{x \to c} x + c + 6$$

$$f'(x) = 2x + 6.$$