INTEGRATION BY SUBSTITUTION

- A substitution of variable can often dramatically simplify an integral.
- When using the method of substitution to evaluate $\int_a^b f(x)dx$ don't forget to also substitute the limits and the increment dx .

291. Evaluate each of the following indefinite integrals by using the suggested substitution:

(a) $\int e^{x^2} x\,dx;\ u = x^2$

(b) $\int x^2\cos(x^3+1)dx;\ u = x^3+1$

(c) $\int \frac{xdx}{x+2};\ p = x+2$

(d) $\int t(t-7)^{12}dt;\ u = t-7$

(e) $\int \frac{dz}{z\ln(z)}dz;\ u = \ln(z)$

(f)(*) $\int \frac{e^x}{1+e^x}dx;\ u = 1+e^x$

292. Evaluate each of the following indefinite integrals:

(a) $\int x(x+3)^9 dx$

(b) $\int e^{\sin(x)}\cos(x)dx$

(c) $\int r\sqrt{1+r^2}dr$

(d)(*) $\int \frac{\sin(\ln(x))dx}{x}$

293. Evaluate each of the following definite integrals:

(a) $\int_0^1 \frac{3x}{(3x+1)^2}dx$

(b) $\int_0^4 e^{x^2+1}x\,dx$

(c) $\int_{\pi/6}^{\pi/4} \frac{\sec^2(x)}{\tan(x)}dx$

(d)(*) $\int_5^{20} \frac{t}{\sqrt{t-4}}dt$

294. (a) Show that $\frac{1}{1-u} + \frac{1}{1+u} = \frac{2}{1-u^2}$.

(b) Hence evaluate $\int \frac{1}{1-u^2}du$.

(c) (*) Evaluate $\int \frac{dx}{(x-5)(7-x)}$ by making the substitution $u = x-6$ and using the result in (b).

295. Find the area of the region bounded by the graph of $y = f(x) = xe^{x^2}$ and the x -axis from $x = 0$ to $x = \sqrt{\ln(8)}$.

296. During a physical examination a patient runs on a treadmill for 3 minutes while his heart rate is monitored. The rate R (in beats per minute) at time t (in minutes) is given by

$$R(t) = 80 + \frac{t(t+1)^4}{10}; \qquad 0 \le t \le 3.$$

(a) What is his heart rate at the beginning and end of the test?

(b) How many times did his heart beat during the examination?

(c) What is his average heart rate during the test?

297. (*) Greg (from Q187) has been released from hospital and is still determined to fill his swimming pool by pumping water from a nearby creek. The pool has a capacity of 1000 m^3 and it currently contains 323 m^3 of water. Greg has purchased the new "*expopump*" which pumps water at a varying rate R (in m^3/min) where

$$R(t) = 1352te^{-t^2}$$

and $t \geq 0$ is time measured in minutes from the turning on of the pump. The pump is disposable and can only be turned on and used once.

(a) At what rate is the water being pumped when $t = 0, 1, 2$ and 3 minutes?

(b) Show that the amount of water in the pool at time t is given by

$$A(t) = 999 - 676e^{-t^2}.$$

(c) When will the pool be 90% full?

(d) When will the pool be full?

298. An individual's blood pressure is being monitored in hospital. Her pressure was initially 100 mm Hg and it subsequently changed at the rate R (in mm Hg/hour) given by

$$R(t) = e^{\sin(t)} \cos(t).$$

(a) Show that her blood pressure is initially increasing but that after 3 hours it is decreasing.

(b) Find a formula for her blood pressure A (in mm Hg) at time t (in hours).

(c) (*) What are the maximum and minimum values for her blood pressure?

SOLUTIONS

Integration by substitution

291. (a) $\frac{1}{2}e^{x^2}+C$ (b) $\frac{\sin(x^3+1)}{3}+C$
(c) $x-2\ln|x+2|+C$ (d) $\frac{(t-7)^{14}}{14}+\frac{7}{13}(t-7)^{13}+C$
(e) $\ln(\ln(z))+C$ (f) $\ln(1+e^x)+C$

292. (a) $\frac{(x+3)^{11}}{11}-\frac{3}{10}(x+3)^{10}+C$ (b) $e^{\sin x}+C$
(c) $\frac{1}{3}(1+r^2)^{3/2}+C$ (d) $-\cos(\ln|x|)+C$

293. (a) $\frac{2}{3}\ln(2)-\frac{1}{4}$ (b) $\frac{1}{2}(e^{17}-e)$ (c) $\frac{1}{2}\ln(3)$ (d) 66

294. (a) Proof. (b) $\frac{1}{2}\ln\left|\frac{1+u}{1-u}\right|+C$ (c) $\frac{1}{2}\ln\left|\frac{x-5}{7-x}\right|+C$

295. $\frac{7}{2}$

296. (a) 80, $\simeq$ 157 (b) 288 (approx.) (c) 96 (approx.)

297. (a) $0, 497.37, 49.53, .5$ (m^3/min) (b) Proof
(c) 1.386 secs (d) The pool will never be full.

298. (a) $R(0)=1>0$ and $R(3)=-1.14<0$ (b) $A=99+e^{\sin(t)}$
(c) 101.72 and 99.37 mm Hg.