3 UNIT – INTEGRATION – WORKSHEET

COURSE/LEVEL

NSW Secondary High School Year 12 HSC Extension Mathematics. Syllabus reference: 11.1 – 11.5

1. Find the primitives of:

   (i) $\sqrt[3]{x^5}$ (i) $\frac{5}{x^3}$

   (iii) $(1-x)^8$ (iv) $\sqrt{3x-1}$

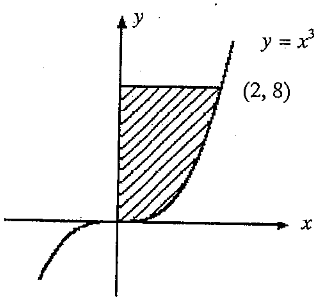
2. Find the area bounded by the curve $y = 3x - x^2$ and the x–axis.

3. Complete the following table:

x	0	1	2	3	4
$f(x)=\frac{1}{x+1}$					

Hence evaluate $\int_0^4 \frac{dx}{x+1}$ using 5 function values of Simpson's Rule.

4. Find the area enclosed between the parabola $y = x^2 + 2x$ and the straight line $y = x$.

5. 

Find the area of the shaded region.

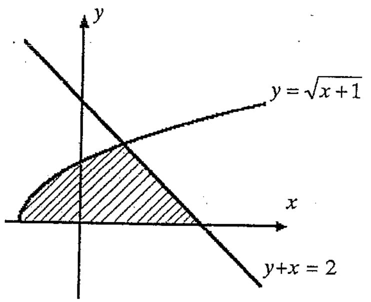
6. If $f''(x) = 6x - 8$ and $f'(0) = 6$, $f(1) = 1$, find $f(x)$.

7. The area bounded by the parabola $y = 9 - x^2$ and the x–axis is rotated about the x–axis. Find the volume generated.

†8. If $y = \sqrt{1-4x^2}$,

   (a) find $\frac{dy}{dx}$.

   (b) Hence evaluate $\int_0^{\frac{1}{2}} \frac{x\,dx}{\sqrt{1-4x^2}}$.

†9. 

Calculate the area of the shaded region.
(Answer to 1 decimal place).

10. Use the substitution $u = x^2 + 1$ to find $\int x(1+x^2)^3\,dx$.

11. By letting $u = 2x + 1$, evaluate $\int_0^4 x\sqrt{2x+1}\,dx$.

12. Use the substitution $t = u^2 - 1$ to evaluate $\int_0^1 \frac{t}{\sqrt{t+1}}\,dt$.

Q1

(i) $y' = x^{\frac{5}{3}}$

$y = \frac{3}{8} x^{8/3} + c$ ✓

(ii) $y' = \frac{5}{x^3}$

$= 5(x)^{-3}$

$y = -\frac{5}{2}(x)^{-2} + c$ ✓

(iii) $y' = (1-x)^8$

$y = -\frac{(1-x)^9}{9} + c$ ✓

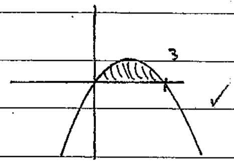
(iv) $y' = \sqrt{3x-1}$

$= (3x-1)^{\frac{1}{2}}$

$y = \frac{2\cdot(3x-1)^{3/2}}{3\times 3} + c$

$= \frac{2}{9}(3x-1)^{3/2} + c$

Q2 $y = 3x - x^2$



$\int_0^3 3x - x^2 \, d$

$= \left[\frac{3}{2}x^2 - \frac{1}{3}x^3\right]_0^3$ ✓

$= (13\tfrac{1}{2} - 9) - 0$

$= 4\tfrac{1}{2}\ u^2$

Q3

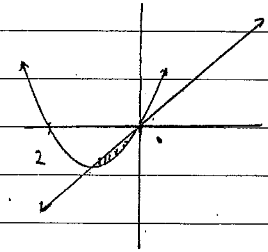
x	0	1	2	3	4
$f(x) = \frac{1}{x+1}$	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$

$\int \frac{dx}{x+1}$

$A = \frac{1}{3}\left[\left(1 + \frac{1}{5}\right) + 4\left(\frac{1}{2} + \frac{1}{4}\right) + 2\left(\frac{1}{3}\right)\right]$ ✓

$= 1\frac{28}{45}\ u^2$ ✓

Q4



$y = x^2 + 2x$

$y = x$

$x = x^2 + 2x$

$0 = x^2 + x$

$= x(x+1)$ ✓

$(-1, -1)$

$\int_{-1}^0 -(x^2 + 2x) + (x)\,dx$

$= -\int_{-1}^0 x^2 + x\,dx$

$= -\left[\frac{1}{3}x^3 + \frac{1}{2}x^2\right]_{-1}^0$ ✓

$= \left(-\frac{1}{3} + \frac{1}{2}\right) = \frac{1}{6}\ u^2$ ✓

Q5

$A_1 = \int_0^2 x^3 \, dx$ $\quad \therefore$ shaded area $=$ area rectangle $- A_1$

$= 16 - 4$

$= 12u^2$ ✓

$= \left[\frac{1}{4}x^4\right]_0^2$

$= 4u^2$ ✓

Q6 $f''(x) = 6x - 8$

$f'(0) = 6$

$f(1) = 1$

$f'(x) = 3x^2 - 8x + c$ ✓

$f'(0) = 3 \cdot 0^2 - 8 \cdot 0 + c$

$f'(0) = 6$

$c = 6$ ✓

$f'(x) = 3x^2 - 8x + 6$ ✓

$f(x) = x^3 - 4x^2 + 6x + c$ ✓

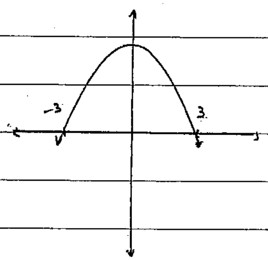
$f(1) = 1$

$f(1) = (1)^3 - 4(1)^2 + 6(1) + c$

$c = -3$ ✓

$f(x) = x^3 - 4x^2 + 6x - 3$ ✓

Q7



$A = \int_{-3}^{3} 9 - x^2 \, dx$ ✓

$2\left[9x - \frac{1}{3}x^3\right]_0^3$

$= 18 \times 2$ ✓

$= 36 u^2$ ✓

Q8 (a) $y = \sqrt{1-4x^2}$

$y = (1-4x^2)^{\frac{1}{2}}$

$y' = \frac{1}{2} \times -8x(1-4x^2)^{-\frac{1}{2}}$

$= -4x(1-4x^2)^{-\frac{1}{2}}$

(b) $\int_0^{\frac{1}{2}} \frac{x}{\sqrt{1-4x^2}} \, dx$ ✓

$= -\frac{1}{4}\int \frac{4x}{\sqrt{1-4x^2}} \, dx$

$= -\frac{1}{4}\left[\sqrt{1-4x^2}\right]_0^{\frac{1}{2}}$

$= \left[0 - \left(-\frac{1}{4}\right)\right]$

$= \frac{1}{4} u^2$ ✓

Q9 $y = \sqrt{x+1}$

$y = 2 - x$

$2 - x = \sqrt{x+1}$

$(2-x)^2 = x + 1$ ✓

$x^2 - 4x + 4 = x + 1$

$x^2 - 5x + 3 = 0$

$x = \frac{5 \pm \sqrt{25 - (4 \cdot 1 \cdot 3)}}{2}$

$= \frac{5 \pm \sqrt{13}}{2}$ ✓

$\int_{-1}^{0.7} \sqrt{x+1} \, dx$

$= \left[\frac{2}{3}(x+1)^{3/2}\right]_{-1}^{0.7}$ ✓

$= 1.477... - 0$ ✓

$= 1.478$ (3 dec. plc.)

$\int_{0.7}^{2} 2 - x \, dx$

$= \int_{0.7}^{2} [2 - x]$

$= \left[2x - \frac{1}{2}x^2\right]_{0.7}^{2}$ ✓

$= 2 - 1.155$ ✓

$= 0.845$

$A_1 + A_2 =$ Total Area

$= 2.323 u^2$ ✓

Q10 $\int x(1+x^2)^3 \, dx$

$\frac{du}{dx} = 2x$

$u = 1 + x^2$ ✓

$\frac{1}{2}\int u^3 \, du$

$= \frac{1}{2}\left[\frac{u^4}{4}\right] + C$ ✓

$= \frac{1}{2} \frac{(1+x^2)^4}{4} + c$

$= \frac{(1+x^2)^4}{8} + c$ ✓

Q11

$u = 2x + 1$ when $x = 4$

$x = \frac{u-1}{2}$ $u = 9$

when $x = 0$

$= \frac{1}{2}\int_0^4 \frac{(u-1)}{2} \cdot u^{\frac{1}{2}}\, du$ $u = 1$

$= \frac{1}{4}\int_1^9 u^{\frac{3}{2}} - u^{\frac{1}{2}}\, du$

$= \frac{1}{4}\left[\frac{2u^{5/2}}{5} - \frac{2u^{3/2}}{3}\right]_1^9$ ✓

$= \frac{1}{4}\left[\frac{486}{5} - 18\right] - \frac{1}{4}\left[\frac{2}{5} - \frac{2}{3}\right]$ ✓

$= 19.8 + \frac{1}{15}$ ✓

$= 19\frac{13}{15}\ u^2$ ✓

Q12. $t = u^2 - 1$

$\frac{dt}{du} = 2u \Rightarrow dt = 2u\, du$

When $t = 0,\ u = 1$; $t = 1,\ u = \sqrt{2}$

$\therefore \int_1^{\sqrt{2}} \frac{u^2-1}{\cancel{2u}} \cdot \cancel{2u}\, du$

$= 2\left[\frac{u^3}{3} - u\right]_1^{\sqrt{2}}$

$= 2\left[\left(\frac{2\sqrt{2}}{3} - \sqrt{2}\right) - \left(\frac{1}{3} - 1\right)\right]$

$= 2\left[-\frac{\sqrt{2}}{3} + \frac{2}{3}\right]$

$= \frac{2}{3}\left[2 - \sqrt{2}\right]$