More difficult problems in Mathematical induction – Ext. 1

1. Prove by mathematical induction that $7^n(3n+1)-1$ is divisible by 9.

Hint: Show that $7^{k+1}(3(k+1)+1)-1-(7^k(3k+1)-1)$ is divisible by 9.

2. Prove by the principle of mathematical induction that:

$$\cos(n\pi + x) = (-1)^n \cos x$$

3. (a) Write down the results for $\sin(x \pm y)$, and hence prove that

$$\sin(k\alpha + \alpha) - \sin(k\alpha - \alpha) = 2\sin\alpha\cos k\alpha .$$

(b) Prove by mathematical induction that

$$\cos\alpha + \cos 3\alpha + \cos 5\alpha + ... + \cos(2n-1)\alpha = \frac{\sin 2n\alpha}{2\sin\alpha}$$

using the result in part (a) at some stage in your proof.