Polynomials 1 + 2

Name: ______________________ Total: /10

Of the three roots of the cubic equation $x^3 - 15x + 4 = 0$, two are reciprocals.

1. Find the other root.

2. Find all the roots and verify that two of them are recipricols.

The cubic polynomial equation $x^3 = ax^2 + bx + c$ has three real roots, two of which are opposites. Prove that

3. One of the roots is a.

4. The other roots are $\sqrt{b}$ and $-\sqrt{b}$.

5. $ab + c = 0$.

$f(x) = x^4 + 4x^3 + 8x - 4$

6. Show that $f(x)$ has a zero, α, between 0 and 1.

7. Determine whether α lies closer to 0 or to 1.

8. Taking 0.5 as a first approximation, use Newton's method to find a two-placed decimal approximation to α.

9. Show by division, or otherwise, that $(x^2 + 2)$ is a factor of $f(x)$.

10. Show that $f(x)$ has only two real roots and find the value of α correct to three decimal places.

Polynomials

1. $x^3 - 15x + 4 = 0$

Let the roots be $\alpha, \frac{1}{\alpha}, \beta$

$$\alpha + \frac{1}{\alpha} + \beta = 0$$

$$1 + \alpha\beta + \frac{\beta}{\alpha} = -15$$

$$\beta = -4 \checkmark$$

2. $1 - 4\alpha - \frac{4}{\alpha} = -15$

$$\alpha - 4\alpha^2 - 4 = -15\alpha$$

$$4\alpha^2 - 16\alpha + 4 = 0 \checkmark$$

$$\alpha^2 - 4\alpha + 1 = 0$$

$$\alpha = \frac{4 \pm \sqrt{16 - 4\cdot 1\cdot 1}}{2} = \frac{4 \pm \sqrt{12}}{2}$$

$$= 2 \pm \sqrt{3} \checkmark$$

One of the roots is $2 + \sqrt{3}$

other root: $\frac{1}{2+\sqrt{3}} \cdot \frac{2-\sqrt{3}}{2-\sqrt{3}}$

$$= \frac{2-\sqrt{3}}{4-3} = 2 - \sqrt{3} \checkmark$$

3. $x^3 - ax^2 - bx - c = 0$

Let the roots be $\alpha, -\alpha, \beta$

$$\alpha - \alpha + \beta = a$$

$$\beta = a \checkmark$$

4. $-\alpha^2 + \alpha\beta - \alpha\beta = -b \checkmark$

$$-\alpha^2 = -b$$

$$\alpha^2 = b$$

$$\alpha = \pm\sqrt{b} \checkmark$$

5. $-\alpha^2\beta = c$

$$-b \cdot a = c \checkmark$$

$$-ab = c$$

$$ab + c = 0 \checkmark$$

6. $f(0) = -4$

$f(1) = 9$

$f(0) < 0$ ✓

$f(1) > 0$

$\therefore \alpha$ lies between 1 & 0 ✓

7. α is closer to 0 ✓

8. $f(x) = x^4 + 4x^3 + 8x - 4$

$f(0.5) = 0.5625$

$f'(x) = 4x^3 + 12x^2 + 8$ ✓

$= 11.5$

$z_2 = 0.5 - \frac{f(0.5)}{f'(0.5)}$

$= 0.45$ ✓

~~1.~~ $\cancel{x^2 + 2 \overline{)\, x^4 + 4x^3 + 8x - 4}}$ with quotient $x^2 + 4x + 2$

$\Delta = b^2 - 4ac$

$= 8$

10b. ~~$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$~~

~~$= \frac{-4 \pm \sqrt{8}}{2}$~~

~~$= -.586$~~ ✓

~~$= -3.41$~~ ✓

(10)(b) For $f(x) = (x^2+2)(x^2+4x-2) = 0$

$x = \frac{-4 \pm \sqrt{16+8}}{2}$

$= 0.449$ or -4.449

$\therefore \alpha = 0.449$ (to 3dp).

$$\begin{array}{r|l} & x^2 + 4x - 2 \\ \hline x^2+0x+2 & x^4 + 4x^3 + 0x^2 + 8x - 4 \\ & x^4 + 0x^3 + 2x^2 \\ \hline & 4x^3 - 2x^2 + 8x \\ & 4x^3 + 0x^2 + 8x \\ \hline & -2x^2 + 0x - 4 \\ & -2x^2 + 0x - 4 \\ \hline & 0 \end{array}$$