POLYNOMIALS 2

1) Show that there is at least one root between $x = 0$ and $x = 1$ of the polynomial $f(x) = 3x^3 - x^2 + 5x - 1$.

2) (a) Show that $p(x) = 5x^2 - 2x - 1$ has a root between $x = 0$ and $x = 1$.
(b) By halving the interval twice, show that the root lies between $x = 0.5$ and $x = 0.75$.
(c) Which of the x values in part (b) is closer to the root? Justify your answer.

3) The function $f(x) = x^2 - \ln(x + 3)$ has a root close to $x = 1$.
(a) Show that the root lies between $x = 1$ and $x = 1.2$
(b) Use the halving the interval method to find the closest value of the root.

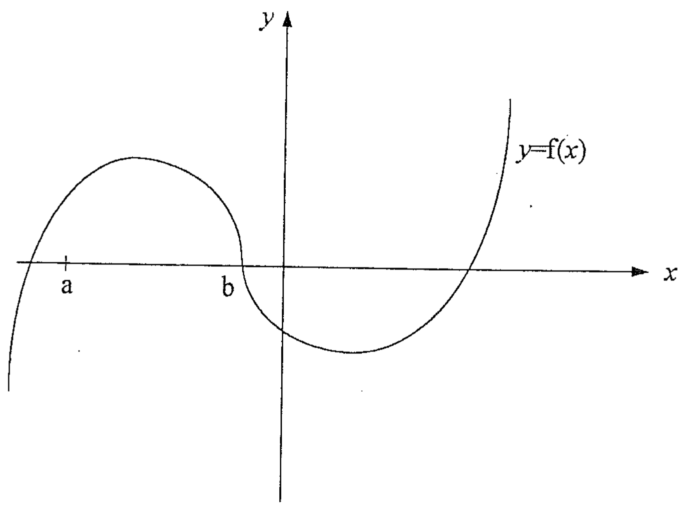
4) Taking $x = 1.9$ as a first approximation to the root of $\frac{x}{2} - \sin x = 0$, use Newton's method to find a second approximation to 4 significant figures.

5) Take $x = 1.1$ as a first approximation to the root of $\frac{x}{4} - \ln x^2$ and use Newton's method to find a second approximation to 3 decimal places.

6) Taking $x = 0.4$ as a first approximation, use Newton's method to find a second approximation to the root of $x + e^x - 2 = 0$ correct to 3 significant figures.

7) Consider the function $f(x) = x^3 - 6x^2 + 9x + 2$.
(a) Show that $f(x) = 0$ has a root between $x = -1$ and $x = 0$.
(b) By halving the interval, show that the root lies between $x = -0.25$ and $x = -0.125$.
(c) By taking the first approximation of the root as $x = -0.25$, use Newton's method to find a second approximation correct to 3 significant figures.

8)



In the graph of $y = f(x)$ above, $x = a$ is taken as the first approximation to the root $x = b$ for $f(x) = 0$. Is the second approximation to b better than a? Give a brief reason for your answer.

9) (a) Show that $f(x) = 16x^2 - 8x - 3$ has a zero between 0 and 1.

(b) Can you use Newton's method with a first approximation of $x = \frac{1}{4}$ to the root of $f(x) = 0$? Why?

(c) Use $x = 0.7$ as a first approximation to find a second approximation to the root using Newton's method. Answer to 3 significant figures.

(d) Find the exact value of the root.

10) Use Newton's method to find an approximation to $\sqrt[3]{30}$ to 3 decimal places. Use $x = 3$ as a first approximation.

ANSWERS

1) f(0) = -1, f(1) = 6 so a root lies between $x = 0$ and $x = 1$.

2) (a) p(0) = -1, p(1) = 2 so root lies between $x = 0$ and $x = 1$.

(b) p(0.5) = -0.75 so root lies between $x = 0.5$ and $x = 1$.

p(0.75) = 0.3125 so root lies between $x = 0.5$ and $x = 0.75$

(c) 0.3125 is closer to 0 than -0.75, so $x = 0.75$ is closer to the root than $x = 0.5$

3) (a) f(1) = -0.386, f(1.2) = 0.0049 so a root lies between $x = 1$ and $x = 1.2$

(b) $x = 1.2$

4) $x = 1.896$

5) $x = 1.154$

6) $x = 0.443$

7) (a) f(-1) = -14, f(0) = 2 so a root lies between $x = -1$ and $x = 0$

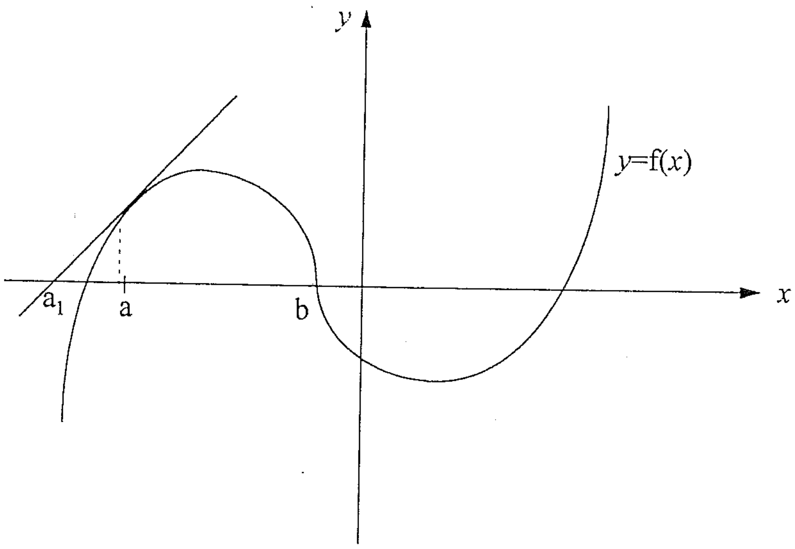
(b) f(-0.5) = -4.125 so root lies between $x = -0.5$ and $x = 0$

f(-0.25) = -0.64 so root lies between $x = -0.25$ and $x = 0$

f(-0.125) = 0.779 so root lies between $x = -0.25$ and $x = -0.125$

(c) $x = -0.197$

8)



$a_1 = a - \frac{f(a)}{f'(a)}$ gives a second approximation further away from b, so the second approximation is not better than a.

9) (a) f(0) = -3, f(1) = 5 so there is a root between $x = 0$ and $x = 1$.

(b) No. $f'(\frac{1}{4}) = 0$ so $a = \frac{1}{4} - \frac{f(\frac{1}{4})}{f'(\frac{1}{4})}$ is undefined.

(c) $x = 0.753$

(d) $x = 0.75$

10) $x = 3.111$