Prove by mathematical induction

1. $\sum_{r=1}^{n}(3r - 1) = \frac{3n^2 + n}{2}$ for all $n \geq 1$
2. $\sum_{r=1}^{n} 2^r = 2(2^n - 1)$ for all $n \geq 1$
3. $\sum_{r=1}^{n} 5r = \frac{5}{2}n(n + 1)$ for all $n \geq 1$
4. $-2 - 4 - 6 - \ldots - 2n = -n(n + 1)$ for all $n \geq 1$
5. $9 + 14 + 19 + \ldots + (5n + 4) = \frac{5n^2 + 13n}{2}$ for all $n \geq 1$
6. $\sum_{r=2}^{n} 3^r = \frac{9(3^{n-1} - 1)}{2}$ for all $n > 1$
7. $4 - 8 + 16 \ldots + 4(-2)^{n-1} = -\frac{4[(-2)^n - 1]}{3}$ for all $n \geq 1$
8. $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{1}{6}n(n + 1)(2n + 1)$
9. $\sum_{r=1}^{n} 3(2^r) = 6(2^n - 1)$
10. $\sum_{r=1}^{n} (4r - 6) = 2n(n - 2)$
11. $\sum_{r=1}^{n} r(r + 1) = \frac{n(n + 1)(n + 2)}{3}$
12. $\sum_{r=1}^{n} (2r - 1)^3 = n^2(2n^2 - 1)$
13. $7^n - 1$ is a multiple of 6 for all positive integers n
14. $3^{2n} - 1$ is divisible by 8 for all $n \geq 1$
15. $5^n - 1$ is divisible by 4 for all $n \geq 1$
16. $5^n + 3^n$ is always even for positive integers n
17. $4^n \geq 3n + 7$ for all integers $n > 1$
18. $5^n - 3 > 4^n + 20$ for $n \geq 3$
19. $n(n + 2)$ is divisible by 4 if n is any even positive integer
20. $7^n + 3^n$ is a multiple of 10 if n is an odd positive integer

14. Solve $|x - 3| + |x + 4| = |x - 2|$.

15. Find the solutions of $x^2 - 2ax - b = 0$ by completing the square.

CHALLENGE EXERCISE 13

1. Prove $a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(r^n - 1)}{r - 1}$ for all a and r, by mathematical induction, where n is a positive integer.
2. Evaluate the sum of the first 10 terms of the series $5 + 8 + 13 + 21 + 34 + 55 + \ldots$.
3. Show $\sum_{r=1}^{n} x^{r-1} = \frac{1 - x^n}{1 - x}$ by mathematical induction.
4. Evaluate the sum of the first 20 terms of the series $3 + 5 + 9 + 17 + 33 + 65 + \ldots$ (hint: $3 = 2 + 1, 5 = 4 + 1$ and so on).
5. A factory sells shoes at \$60 each. For 10 pairs of shoes there is a discount, whereby each pair costs \$58. For 20 pairs, the price of each pair is \$56, and so on. Find
   (a) the price of each pair of shoes on an order of 100 pairs
   (b) the total price of an order of 60 pairs of shoes
6. (a) Evaluate $\sum_{n=1}^{25} 100 - 3n$.
   (b) Prove $\sum_{r=1}^{n} 100 - 3r = \frac{n}{2}(197 - 3n)$ by mathematical induction.
7. Find the sum of all integers between 1 and 200 that are not multiples of 7.