South Sydney High School
RATES OF CHANGE
3 Unit Worksheet

1. A block of ice, mass 4.5 kg, is placed in the sun to melt. After t minutes the mass of ice remaining is M kg, where

$$M = 4.5 - \frac{t^2}{200}.$$

(a) Find the time taken for all the ice to melt.

(b) Find the rate at which the ice is melting

(i) after 15 minutes

(ii) when the mass remaining is 2.5 kg.

2. Water runs into a container, originally empty, so that, after t minutes, it is running at R litres/minute where

$$R = (0.8)t$$

(i.e. when the volume of water in the container is V litres, $\frac{dV}{dt} = (0.8)t$)

(a) How much water will enter in the first 5 minutes ?

(b) How long will it take to fill the container if its capacity is 90 litres ?

3. A tank is emptied of water by a continually opening valve so that, t minutes after the valve begins to open, the volume, V litres, of water in the tank is given by

$V = a + bt^2$, where a and b are constants.

Initially the tank contains 500 litres and it takes 5 minutes to empty it.

(a) Find the values of a and b.

(b) Find the rate at which the volume is changing

(i) after 2.5 minutes

(ii) when the tank is half empty (to the nearest litre/minute)

4. A square metal plate has sides of x cm and area A cm^2. It is expanding so that the sides are increasing at 0.08 cm/min. (i.e. $\frac{dx}{dt} = 0.08$). Find the rate at which the area is increasing (i.e. $\frac{dA}{dt}$)

(a) when the sides are 7 cm long

(b) when the area is 100 cm^2

5. A circle is expanding so that the rate of increase in its radius is 0.75 cm/s. Find the rate of increase in its area when

(a) the radius is 10 cm

(b) the circumference is 10 cm

(c) the area is 10 cm^2

6. A metal cube has sides of x cm and volume of V cm^3. The cube is cooling so that the lengths of its sides are decreasing at 0.075 cm/min. (i.e. $\frac{dx}{dt} = -0.075$). Find the rate of change in its volume

(a) when the sides are 4 cm long

(b) when the total surface area is 100 cm^2

(c) when the volume is 1000 cm^3

7. For the cube in Question 6, find the rate of change of its surface area when the area is 150 cm^2

8. A tank is emptied by a tap from which the water flows so that, until the flow ceases, the rate after t minutes is R litres/minute where

$$R = (t - 6)^2$$

(a) What is the initial rate of flow ?

(b) How long does it take to empty the tank ?

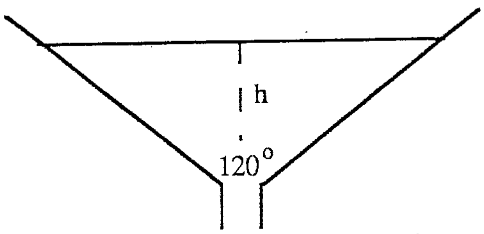
(c) How long will it take (to the nearest second) for the flow to drop to 20 litres/minute ?

(d) How much water was in the tank initially ?

9. Air is pumped into a balloon at a constant rate of 15 cm^3/s. Find the rate of increase in its surface area when its radius is 10 cm.

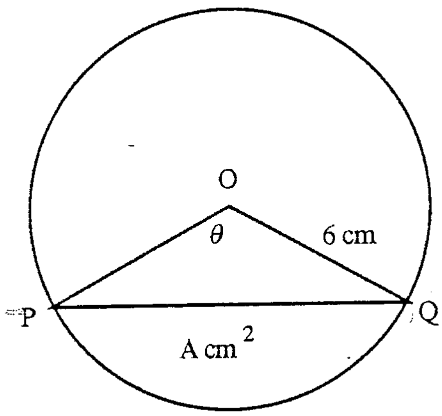
(Note: $\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dV} \cdot \frac{dV}{dt}$)

10.



A filter funnel with a vertical angle of 120° contains liquid to a depth of h cm. Find expressions for the volume of the liquid and its horizontal surface area. If the volume is decreasing at 30 ml/min., at what rate is the height decreasing when the surface area is 40 cm^2 ?

11.



O is the centre of the circle, radius 6 cm and $\angle$ POQ contains θ radians.

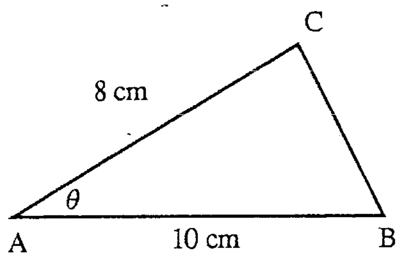
(a) Find an expression for A, the area of the segment, in terms of θ.

(b) If θ is increasing at 0.75 radians/second, what is the rate of change of A

(i) when $\theta = \frac{\pi}{3}$ (ii) when $\theta = \frac{2\pi}{3}$

† 12. A certain species of tree grows so that, at any time, its height is increasing at $\frac{A}{2t + 1}$ m/year, where A is a constant depending on climate and soil conditions. In a new area, one such tree, planted at a height of 1 m, reaches a height of 3 m after 1 year. Estimate its height 10 years after planting.

† 13.

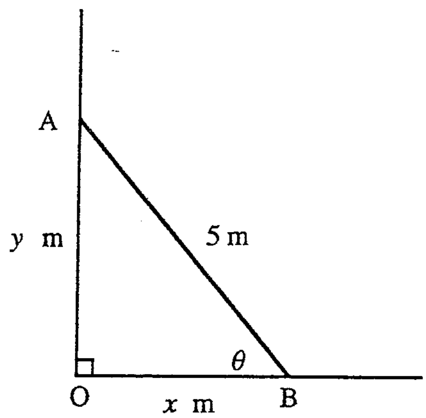


$\angle$ BAC is increasing at a constant rate of 0.2 radians/second. Find the rate of change in the area of Δ ABC

(a) when $\theta = \frac{\pi}{3}$ (b) when $\theta = \frac{2\pi}{3}$

(c) on each of the two occasions when Δ ABC is right-angled.

† 14.



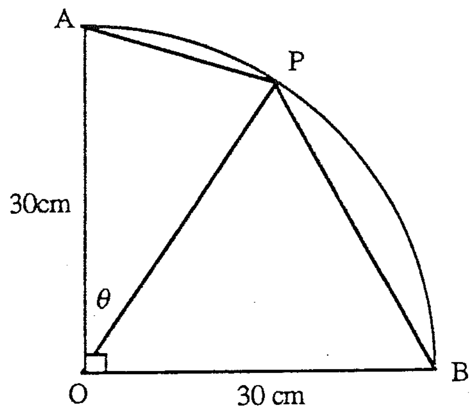
A ladder, AB, 5 m long, stands on horizontal ground against a vertical wall. The base, B, is then drawn away from the wall at a constant rate of 0.5 m/s.

(a) Prove that $\frac{dx}{d\theta} = -5\sin\theta$

(b) Find the rate at which θ is changing when $x = 3$

(c) Find the rate at which A is moving when $x = 3$

† 15.



AOB is a quadrant of a circle. P rotates about O at a constant rate, moving from A to B in 15 minutes. If S is the total area of Δ OAP and Δ OBP, find the rate at which S is changing when $\theta = \dfrac{\pi}{6}$.

RATES OF CHANGE

1. (a) 30 m (b) (i) 0.15 kg/m (ii) 0.2 kg/m
2. (a) 10 L (b) 15 m
3. (a) $a = 500$, $b = -20$
   (b) (i) 100 L/m (ii) 141 L/m
4. (a) 1.12 cm^2/m (b) 1.6 cm^2/m
5. (a) 15 π cm^2/s (b) 7.5 cm^2/s (c) $\dfrac{3\sqrt{10\pi}}{2}$ cm^2/s
6. (a) 3.6 cm^3/m (b) 3.75 cm^3/m (c) 22.5 cm^3/m
7. 4.5 cm^2/m
8. (a) 36 L/m (b) 6 m (c) 1 m 32 s (d) 72 L
9. 3 cm^2/s
10. $V = \pi h^3$; $A = 3\pi h^2$; 0.75 cm/m
11. (a) $18(\theta - \sin\theta)$
    (b) (i) 6.75 cm^2/s (ii) 20.25 cm^2/s
12. 6.54 m
13. (a) 4 cm^2/s (b) – 4 cm^2/s (c) 6.4 cm^2/s and 0 cm^2/s
14. (b) 0.125 rad/s (c) 0.375 m/s
15. $\dfrac{15\pi(\sqrt{3}-1)}{2}$ cm^2/m

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