CRANBROOK SCHOOL

YEAR 12 EXT1 MATHEMATICS – TEST

16th February, 2006 Circle teacher: **JSH HRK BMM SKB**

- **Geometrical Applications of Calculus**
- **Newton's Method** **Time: 50mins**

All necessary working should be shown in every question.
Full marks may not be awarded if work is careless or badly arranged.
Approved silent calculators may be used.
Begin each question on a new page.

1. **(12 marks) (Begin a new page)** **JSH**

The curve $f(x) = x^3 + 3x^2 - 9x - 1$ is defined in the domain $-4 \le x \le 2$.

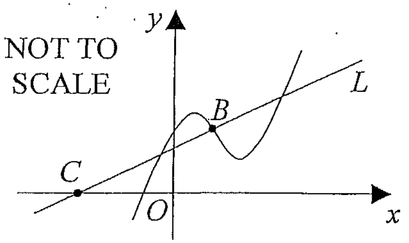
i. Find the co-ordinates of the two stationary points and determine their nature.

ii. Show a point of inflexion occurs at $x = -1$.

iii. Sketch this curve

2. **(12 marks) (Begin a new page)** **JSH**

NOT TO SCALE



The diagram shows a sketch of the curve $y = x^3 - 6x^2 + 9x + 4$. The curve has a point of inflexion at B. The line L is a normal to the curve at point B and meets the x axis at a point C.

(i) Show that the coordinates of point B is $(2, 6)$.

(ii) Show that the equation of the line L is $x - 3y + 16 = 0$.

(iii) Find the coordinates of point C.

(iv)

3. **(12 marks) (Begin a new page)** **HRK**

A closed water tank in the shape of a right cylinder is to be constructed with a surface area of 54π cm^2. The height of the cylinder is h cm and the base radius is r cm.

i. Show that height of the water tank in terms of r is given by $h = \frac{27}{r} - r$.

ii. Show that the volume V that can be contained in the tank is given by $V = 27\pi r - \pi r^3$.

iii. Find the radius r cm which will give the cylinder its greatest possible volume. Justify your answer.

4. **(12 marks) (Begin a new page)** **HRK**

Given the polynomial equation $f(x) = 8x^3 + 12x^2 - 18x - 20 = 0$

a. Show that it has a root between $x = 1$ and $x = 2$

b. Use the method of halving the interval twice to find an approximation to this root of the equation

c. Apply Newton's method once to approximate this root

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(1) $f(x) = x^3 + 3x^2 - 9x - 1, \; -4 \le x \le 2$

(i) $f'(x) = 3x^2 + 6x - 9$
$f''(x) = 6x + 6$ ✓

For a stat pt $f'(x) = 0$
$\therefore 3(x^2 + 2x - 3) = 0$
$\therefore 3(x+3)(x-1) = 0$ ✓
$\therefore x = -3$ or 1

✓✓ when $x = -3$ $f''(-3) < 0 \Rightarrow$ max turn pt at $(-3, 26)$

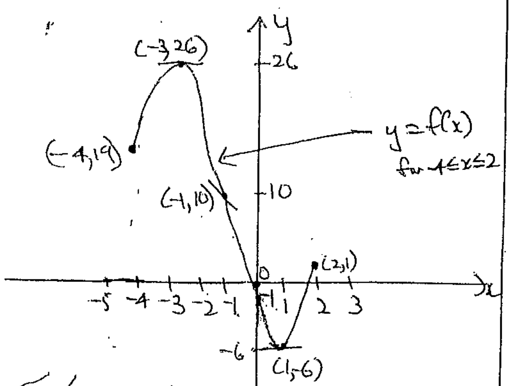
✓✓ $x = 1$ $f''(1) > 0 \Rightarrow$ min turn pt at $(1, -6)$

(ii) For a possible pt of inflexion
$f''(x) = 0 \quad \therefore x = -1$ ✓

x	-1^-	-1	-1^+
$f''(x)$	$-$	0	$+$

$\underbrace{\qquad\qquad}_{\text{concavity change}}$ ✓
$\Rightarrow$ pt of inflexion at $(-1, 10)$ ✓

(iii)



✓ end points
✓ y-intercept

(2) $y = x^3 - 6x^2 + 9x + 4$
$y' = 3x^2 - 12x + 9$ ✓
$y'' = 6x - 12$ ✓

(i) As there is a pt of inflexion at B
$\therefore$ at B $y'' = 0 \quad \therefore x = 2$ ✓✓
when $x = 2$ $y = 2^3 - 6(2)^2 + 9(2) + 4 = 6$ ✓
$\Rightarrow$ B has co-ords $(2, 6)$

(ii) at $B(2,6)$ $y' = 3(2)^2 - 12(2) + 9$
$= -3 \quad = m_{tang}$ ✓
$\therefore m_{norm} = \frac{1}{3}$ ✓
$\therefore$ eqn of line L is: $y - 6 = \frac{1}{3}(x - 2)$ ✓
$\therefore 3y - 18 = x - 2$
$\therefore x - 3y + 16 = 0$ ✓

(iii) At C $y = 0 \quad \therefore x = -16$
$\Rightarrow$ coords of C are: $(-16, 0)$ ✓✓

(3) $V = \pi r^2 h$ —(1) ✓
$A = 2\pi r^2 + 2\pi r h = 54\pi$ —(2) ✓

(i) from (2) $h = \dfrac{54\pi - 2\pi r^2}{2\pi r}$ ✓
$\therefore h = \dfrac{27}{r} - r$ —(3) ✓

(ii) sub (3) into (1)
$\therefore V = \pi r^2 \left(\dfrac{27}{r} - r\right)$
$\therefore V = 27\pi r - \pi r^3$ ✓✓

(iii) $V' = 27\pi - 3\pi r^2$ ✓
$V'' = -6\pi r$ ✓

For a possible max/min $V' = 0$
$\therefore r^2 = \dfrac{27\pi}{3\pi} = 9$
$\therefore r = 3$ ($r > 0$ as radius must be ✓✓

3(i)

$SA = 2\pi r^2 + 2\pi r h$ ✓

$54\cancel{\pi} = 2\cancel{\pi} r^2 + 2\cancel{\pi} r h$

$\dfrac{2rh}{2r} = \dfrac{54 - 2r^2}{2r}$ ✓

$h = \dfrac{27}{r} - r$ ✓

(ii) $V = \pi r^2 h$ ✓

$= \pi r^2 \left(\dfrac{27}{r} - r\right)$ ✓

$= 27\pi r - \pi r^3$ ✓

(iii) $V = 2\ r - \pi r^3$ ✓

$V' = 2\ \pi - 3\pi r^2$ ✓

$V' = 0$ FOR MAX

$27\cancel{\pi} = 3\cancel{\pi} r^2$

$r = \pm 3, \; r > 0$ ✓

$\therefore r = 3$

$V'' = -6\pi r$ ✓

$= -18\pi$ when $r = 3$

< 0 ∩ ✓

$\therefore$ MAX Volume occurs when $r = 3$ cm ✓

4(a)

$f(1) = -18$ ✓

$f(2) = 56$ ✓

Since $f(x)$ is continuous and $f(1)$ & $f(2)$ have opposite signs there is at least 1 root in the interval $1 < x < 2$ ✓

b) $x_1 = \frac{1+2}{2} = 1.5$ ✓

$f(1.5) = 7$ ✓

Since $f(x)$ is cts and $f(1.5)$ and $f(1)$ have opposite signs there is at least 1 root for $1 < x < 1.5$ ✓

$x_2 = \frac{1 + 1.5}{2} = 1.25$ ✓

c) $f'(x) = 24x^2 + 24x - 18$ ✓

$z_2 = z_1 - \dfrac{f(z_1)}{f'(z_1)}$ ✓

$= 1.5 - \dfrac{7}{72}$ ✓✓

$= 1.40$ (2dp) ✓